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Complex Numbers and Quadratic Equations



TOPIC 1

Integral Powers of i , Algebraic Operations of Complex Numbers, Conjugate, Modulus and Argument or Amplitude of a Complex Number



- If $\frac{3+i\sin\theta}{4-i\cos\theta}$, $\theta \in [0, 2\pi]$, is a real number, then an argument of $\sin\theta + i\cos\theta$ is: **[Jan. 7, 2020 (II)]**

(a) $\pi - \tan^{-1}\left(\frac{4}{3}\right)$ (b) $\pi - \tan^{-1}\left(\frac{3}{4}\right)$
 (c) $-\tan^{-1}\left(\frac{3}{4}\right)$ (d) $\tan^{-1}\left(\frac{4}{3}\right)$
- If the four complex numbers $z, \bar{z}, \bar{z} - 2\operatorname{Re}(\bar{z})$ and $z - 2\operatorname{Re}(z)$ represent the vertices of a square of side 4 units in the Argand plane, then $|z|$ is equal to: **[Sep. 05, 2020 (I)]**

(a) $4\sqrt{2}$ (b) 4 (c) $2\sqrt{2}$ (d) 2
- The value of $\left(\frac{-1+i\sqrt{3}}{1-i}\right)^{30}$ is: **[Sep. 05, 2020 (II)]**

(a) -2^{15} (b) $2^{15}i$ (c) $-2^{15}i$ (d) 6^5
- If $\left(\frac{1+i}{1-i}\right)^{m/2} = \left(\frac{1+i}{i-1}\right)^{n/3} = 1$, ($m, n \in \mathbf{N}$), then the greatest common divisor of the least values of m and n is: **[Sep. 03, 2020 (I)]**
- If z_1, z_2 are complex numbers such that $\operatorname{Re}(z_1) = |z_1 - 1|$, $\operatorname{Re}(z_2) = |z_2 - 1|$ and $\arg(z_1 - z_2) = \frac{\pi}{6}$, then $\operatorname{Im}(z_1 + z_2)$ is equal to: **[Sep. 03, 2020 (II)]**

(a) $\frac{2}{\sqrt{3}}$ (b) $2\sqrt{3}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{3}}$
- Let z be a complex number such that $\left|\frac{z-i}{z+2i}\right| = 1$ and $|z| = \frac{5}{2}$. Then the value of $|z + 3i|$ is: **[Jan. 9, 2020 (I)]**

(a) $\sqrt{10}$ (b) $\frac{7}{2}$ (c) $\frac{15}{4}$ (d) $2\sqrt{3}$
- If z be a complex number satisfying $|\operatorname{Re}(z)| + |\operatorname{Im}(z)| = 4$, then $|z|$ cannot be: **[Jan. 9, 2020 (II)]**

(a) $\sqrt{\frac{17}{2}}$ (b) $\sqrt{10}$ (c) $\sqrt{7}$ (d) $\sqrt{8}$
- Let $z \in \mathbf{C}$ with $\operatorname{Im}(z) = 10$ and it satisfies $\frac{2z-n}{2z+n} = 2i - 1$ for some natural number n . Then: **[April 12, 2019 (II)]**

(a) $n = 20$ and $\operatorname{Re}(z) = -10$
 (b) $n = 40$ and $\operatorname{Re}(z) = 10$
 (c) $n = 40$ and $\operatorname{Re}(z) = -10$
 (d) $n = 20$ and $\operatorname{Re}(z) = 10$
- The equation $|z - i| = |z - 1|$, $i = \sqrt{-1}$, represents: **[April 12, 2019 (I)]**

(a) a circle of radius $\frac{1}{2}$.
 (b) the line through the origin with slope 1.
 (c) a circle of radius 1.
 (d) the line through the origin with slope -1 .
- If $a > 0$ and $z = \frac{(1+i)^2}{a-i}$, has magnitude $\sqrt{\frac{2}{5}}$, then \bar{z} is equal to: **[April 10, 2019 (I)]**

(a) $-\frac{1}{5} - \frac{3}{5}i$ (b) $-\frac{3}{5} - \frac{1}{5}i$
 (c) $\frac{1}{5} - \frac{3}{5}i$ (d) $-\frac{1}{5} + \frac{3}{5}i$

11. If z and ω are two complex numbers such that $|z\omega|=1$ and $\arg(z) - \arg(\omega) = \frac{\pi}{2}$, then: **[April 10, 2019 (II)]**
- (a) $\bar{z}\omega = i$ (b) $z\bar{\omega} = \frac{-1+i}{\sqrt{2}}$
 (c) $\bar{z}\omega = -i$ (d) $z\bar{\omega} = \frac{1-i}{\sqrt{2}}$
12. Let $z \in \mathbb{C}$ be such that $|z| < 1$. If $\bar{\omega} = \frac{5+3z}{5(1-z)}$, then : **[April 09, 2019 (II)]**
- (a) $5 \operatorname{Re}(\omega) > 4$ (b) $4 \operatorname{Im}(\omega) > 5$
 (c) $5 \operatorname{Re}(\omega) > 1$ (d) $5 \operatorname{Im}(\omega) < 1$
13. If $\frac{z-\alpha}{z+\alpha}$ ($\alpha \in \mathbb{R}$) is a purely imaginary number and $|z|=2$, then a value of α is : **[Jan. 12, 2019 (I)]**
- (a) 2 (b) 1 (c) $\frac{1}{2}$ (d) $\sqrt{2}$
14. Let z_1 and z_2 be two complex numbers satisfying $|z_1|=9$ and $|z_2| - |3|-|4i|=|4|$. Then the minimum value of $|z_1 - z_2|$ is : **[Jan. 12, 2019 (II)]**
- (a) 0 (b) $\sqrt{2}$ (c) 1 (d) 2
15. Let z be a complex number such that $|z| + z = 3 + i$ (where $i = \sqrt{-1}$). Then $|z|$ is equal to : **[Jan. 11, 2019 (II)]**
- (a) $\frac{\sqrt{34}}{3}$ (b) $\frac{5}{3}$ (c) $\frac{\sqrt{41}}{4}$ (d) $\frac{5}{4}$
16. Let z_1 and z_2 be any two non-zero complex numbers such that $3|z_1| = 4|z_2|$. If $z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1}$ then: **[Jan. 10 2019 (II)]**
- (a) $\operatorname{Re}(z) = 0$ (b) $|z| = \sqrt{\frac{5}{2}}$
 (c) $|z| = \frac{1}{2}\sqrt{\frac{17}{2}}$ (d) $\operatorname{Im}(z) = 0$
17. Let $A = \left\{ \theta \in \left(-\frac{\pi}{2}, \pi \right) : \frac{3+2i\sin\theta}{1-2i\sin\theta} \text{ is purely imaginary} \right\}$. Then the sum of the elements in A is: **[Jan. 9 2019 (I)]**
- (a) $\frac{5\pi}{6}$ (b) π (c) $\frac{3\pi}{4}$ (d) $\frac{2\pi}{3}$
18. The set of all $\alpha \in \mathbb{R}$, for which $w = \frac{1+(1-8\alpha)z}{1-z}$ is a purely imaginary number, for all $z \in \mathbb{C}$ satisfying $|z|=1$ and $\operatorname{Re} z \neq 1$, is **[Online April 15, 2018]**
- (a) $\{0\}$ (b) an empty set
 (c) $\left\{ 0, \frac{1}{4}, -\frac{1}{4} \right\}$ (d) equal to \mathbb{R}
19. A value of θ for which $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, is: **[2016]**
- (a) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ (b) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$
 (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$
20. If z is a non-real complex number, then the minimum value of $\frac{\operatorname{Im} z^5}{(\operatorname{Im} z)^5}$ is : **[Online April 11, 2015]**
- (a) -1 (b) -4 (c) -2 (d) -5
21. If z is a complex number such that $|z| \geq 2$, then the minimum value of $\left| z + \frac{1}{z} \right|$: **[2014]**
- (a) is strictly greater than $\frac{5}{2}$
 (b) is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$
 (c) is equal to $\frac{5}{2}$
 (d) lie in the interval $(1, 2)$
22. For all complex numbers z of the form $1 + i\alpha$, $\alpha \in \mathbb{R}$, if $z^2 = x + iy$, then **[Online April 19, 2014]**
- (a) $y^2 - 4x + 2 = 0$ (b) $y^2 + 4x - 4 = 0$
 (c) $y^2 - 4x - 4 = 0$ (d) $y^2 + 4x + 2 = 0$
23. Let $z \neq -i$ be any complex number such that $\frac{z-i}{z+i}$ is a purely imaginary number. Then $z + \frac{1}{z}$ is: **[Online April 12, 2014]**
- (a) zero
 (b) any non-zero real number other than 1.
 (c) any non-zero real number.
 (d) a purely imaginary number.



24. If z_1, z_2 and z_3, z_4 are 2 pairs of complex conjugate numbers, then

$\arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right)$ equals: **[Online April 11, 2014]**

- (a) 0 (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{2}$ (d) π

25. Let w ($\text{Im } w \neq 0$) be a complex number. Then the set of all complex number z satisfying the equation

$w - \bar{w}z = k(1 - z)$, for some real number k , is

[Online April 9, 2014]

- (a) $\{z : |z| = 1\}$ (b) $\{z : z = \bar{z}\}$
 (c) $\{z : z \neq 1\}$ (d) $\{z : |z| = 1, z \neq 1\}$

26. If z is a complex number of unit modulus and

argument θ , then $\arg\left(\frac{1+z}{1+\bar{z}}\right)$ equals: **[2013]**

- (a) $-\theta$ (b) $\frac{\pi}{2} - \theta$ (c) θ (d) $\pi - \theta$

27. Let z satisfy $|z| = 1$ and $z = 1 - \bar{z}$.

Statement 1 : z is a real number.

Statement 2 : Principal argument of z is $\frac{\pi}{3}$

[Online April 25, 2013]

- (a) Statement 1 is true Statement 2 is true; Statement 2 is a correct explanation for Statement 1.
 (b) Statement 1 is false; Statement 2 is true
 (c) Statement 1 is true, Statement 2 is false.
 (d) Statement 1 is true; Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.

28. Let $a = \text{Im}\left(\frac{1+z^2}{2iz}\right)$, where z is any non-zero complex number.

[Online April 23, 2013]

The set $A = \{a : |z| = 1 \text{ and } z \neq \pm 1\}$ is equal to:

- (a) $(-1, 1)$ (b) $[-1, 1]$ (c) $[0, 1)$ (d) $(-1, 0]$

29. If $Z_1 \neq 0$ and Z_2 be two complex numbers such that $\frac{Z_2}{Z_1}$

is a purely imaginary number, then $\left|\frac{2Z_1 + 3Z_2}{2Z_1 - 3Z_2}\right|$ is equal to:

[Online April 9, 2013]

- (a) 2 (b) 5 (c) 3 (d) 1

30. $|z_1 + z_2|^2 + |z_1 - z_2|^2$ is equal to **[Online May 26, 2012]**

- (a) $2(|z_1| + |z_2|)$ (b) $2(|z_1|^2 + |z_2|^2)$
 (c) $|z_1||z_2|$ (d) $|z_1|^2 + |z_2|^2$

31. Let Z and W be complex numbers such that $|Z| = |W|$, and $\arg Z$ denotes the principal argument of Z .

[Online May 19, 2012]

Statement 1: If $\arg Z + \arg W = \pi$, then $Z = -\bar{W}$.

Statement 2: $|Z| = |W|$, implies $\arg Z - \arg \bar{W} = \pi$.

- (a) Statement 1 is true, Statement 2 is false.
 (b) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.
 (c) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1.
 (d) Statement 1 is false, Statement 2 is true.

32. Let Z_1 and Z_2 be any two complex number.

Statement 1: $|Z_1 - Z_2| \geq |Z_1| - |Z_2|$

Statement 2: $|Z_1 + Z_2| \leq |Z_1| + |Z_2|$ **[Online May 7, 2012]**

- (a) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation of Statement 1.
 (b) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation of Statement 1.
 (c) Statement 1 is true, Statement 2 is false.
 (d) Statement 1 is false, Statement 2 is true.

33. The number of complex numbers z such that $|z - 1| = |z + 1| = |z - i|$ equals **[2010]**

- (a) 1 (b) 2 (c) ∞ (d) 0

34. The conjugate of a complex number is $\frac{1}{i-1}$ then that complex number is **[2008]**

- (a) $\frac{-1}{i-1}$ (b) $\frac{1}{i+1}$ (c) $\frac{-1}{i+1}$ (d) $\frac{1}{i-1}$

35. If $z = x - iy$ and $\frac{1}{z^3} = p + iq$, then $\left(\frac{x}{p} + \frac{y}{q}\right) / (p^2 + q^2)$ is equal to **[2004]**

- (a) -2 (b) -1 (c) 2 (d) 1

36. Let z and w be complex numbers such that $\bar{z} + i\bar{w} = 0$ and $\arg zw = \pi$. Then $\arg z$ equals **[2004]**

- (a) $\frac{5\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{4}$ (d) $\frac{\pi}{4}$

37. If $\left(\frac{1+i}{1-i}\right)^x = 1$ then **[2003]**

- (a) $x = 2n + 1$, where n is any positive integer
 (b) $x = 4n$, where n is any positive integer
 (c) $x = 2n$, where n is any positive integer
 (d) $x = 4n + 1$, where n is any positive integer.

38. If z and ω are two non-zero complex numbers such that $|z\omega| = 1$ and $\text{Arg}(z) - \text{Arg}(\omega) = \frac{\pi}{2}$, then $\bar{z}\omega$ is equal to [2003]
 (a) -1 (b) 1 (c) $-i$ (d) i
39. If $|z-4| < |z-2|$, its solution is given by [2002]
 (a) $\text{Re}(z) > 0$ (b) $\text{Re}(z) < 0$
 (c) $\text{Re}(z) > 3$ (d) $\text{Re}(z) > 2$
40. z and w are two non zero complex numbers such that $|z| = |w|$ and $\text{Arg } z + \text{Arg } w = \pi$ then z equals [2002]
 (a) $\bar{\omega}$ (b) $-\bar{\omega}$ (c) ω (d) $-\omega$
- TOPIC 2** **Rotational Theorem, Square Root of a Complex Number, Cube Roots of Unity, Geometry of Complex Numbers, De-moiver's Theorem, Powers of Complex Numbers**
41. Let $z = x + iy$ be a non-zero complex number such that $z^2 = i|z|^2$, where $i = \sqrt{-1}$, then z lies on the: [Sep. 06, 2020 (II)]
 (a) line, $y = -x$ (b) imaginary axis
 (c) line, $y = x$ (d) real axis
42. If a and b are real numbers such that $(2 + \alpha)^4 = a + b\alpha$, where $\alpha = \frac{-1 + i\sqrt{3}}{2}$, then $a + b$ is equal to : [Sep. 04, 2020 (II)]
 (a) 9 (b) 24 (c) 33 (d) 57
43. The value of $\left(\frac{1 + \sin \frac{2\pi}{9} + i \cos \frac{2\pi}{9}}{1 + \sin \frac{2\pi}{9} - i \cos \frac{2\pi}{9}} \right)^3$ is : [Sep. 02, 2020 (I)]
 (a) $\frac{1}{2}(1 - i\sqrt{3})$ (b) $\frac{1}{2}(\sqrt{3} - i)$
 (c) $-\frac{1}{2}(\sqrt{3} - i)$ (d) $-\frac{1}{2}(1 - i\sqrt{3})$
44. The imaginary part of $(3 + 2\sqrt{-54})^{1/2} - (3 - 2\sqrt{-54})^{1/2}$ can be : [Sep. 02, 2020 (II)]
 (a) $-\sqrt{6}$ (b) $-2\sqrt{6}$ (c) 6 (d) $\sqrt{6}$
45. Let $\alpha = \frac{-1 + i\sqrt{3}}{2}$. If $a = (1 + \alpha) \sum_{k=0}^{100} \alpha^{2k}$ and $b = \sum_{k=0}^{100} \alpha^{3k}$, then a and b are the roots of the quadratic equation: [Jan. 8, 2020 (II)]
 (a) $x^2 + 101x + 100 = 0$ (b) $x^2 - 102x + 101 = 0$
 (c) $x^2 - 101x + 100 = 0$ (d) $x^2 + 102x + 101 = 0$
46. If $\text{Re} \left(\frac{z-1}{2z+i} \right) = 1$, where $z = x + iy$, then the point (x, y) lies on a: [Jan. 7, 2020 (I)]
 (a) circle whose centre is at $\left(-\frac{1}{2}, -\frac{3}{2} \right)$.
 (b) straight line whose slope is $-\frac{2}{3}$.
 (c) straight line whose slope is $\frac{3}{2}$.
 (d) circle whose diameter is $\frac{\sqrt{5}}{2}$.
47. If $z = \frac{\sqrt{3}}{2} + \frac{i}{2}(i = \sqrt{-1})$, then $(1 + iz + z^5 + iz^8)^9$ is equal to: [April 08, 2019 (II)]
 (a) 0 (b) 1
 (c) $(-1 + 2i)^9$ (d) -1
48. Let $\left(-2 - \frac{1}{3}i \right)^3 = \frac{x + iy}{27}(i = \sqrt{-1})$, where x and y are real numbers then $y - x$ equals : [Jan. 11, 2019 (I)]
 (a) 91 (b) -85 (c) 85 (d) -91
49. Let $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2} \right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2} \right)^5$. If $R(z)$ and $I(z)$ respectively denote the real and imaginary parts of z , then: [Jan. 10, 2019 (II)]
 (a) $I(z) = 0$ (b) $R(z) > 0$ and $I(z) > 0$
 (c) $R(z) < 0$ and $I(z) > 0$ (d) $R(z) = -I(z)$
50. The least positive integer n for which $\left(\frac{1 + i\sqrt{3}}{1 - i\sqrt{3}} \right)^n = 1$, is [Online April 16, 2018]
 (a) 2 (b) 6 (c) 5 (d) 3
51. The point represented by $2 + i$ in the Argand plane moves 1 unit eastwards, then 2 units northwards and finally from there $2\sqrt{2}$ units in the south-westwards direction. Then its new position in the Argand plane is at the point represented by : [Online April 9, 2016]
 (a) $1 + i$ (b) $2 + 2i$ (c) $-2 - 2i$ (d) $-1 - i$
52. A complex number z is said to be unimodular if $|z| = 1$. Suppose z_1 and z_2 are complex numbers such that $\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a: [2015]

- (a) circle of radius 2.
- (b) circle of radius $\sqrt{2}$.
- (c) straight line parallel to x-axis
- (d) straight line parallel to y-axis.

53. If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies : [2012]

- (a) either on the real axis or on a circle passing through the origin.
- (b) on a circle with centre at the origin
- (c) either on the real axis or on a circle not passing through the origin.
- (d) on the imaginary axis.

54. If $\omega (\neq 1)$ is a cube root of unity, and $(1+\omega)^7 = A+B\omega$. Then (A, B) equals [2011]

- (a) (1, 1) (b) (1, 0) (c) (-1, 1) (d) (0, 1)

55. If $|z + 4| \leq 3$, then the maximum value of $|z + 1|$ is [2007]

- (a) 6 (b) 0 (c) 4 (d) 10

56. If $\omega = \frac{z}{z - \frac{1}{3}i}$ and $|\omega| = 1$, then z lies on [2005]

- (a) an ellipse (b) a circle
- (c) a straight line (d) a parabola

57. If z_1 and z_2 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then $\arg z_1 - \arg z_2$ is equal to [2005]

- (a) $\frac{\pi}{2}$ (b) $-\pi$ (c) 0 (d) $-\frac{\pi}{2}$

58. If the cube roots of unity are $1, \omega, \omega^2$ then the roots of the equation $(x-1)^3 + 8 = 0$, are [2005]

- (a) $-1, -1 + 2\omega, -1 - 2\omega^2$
- (b) $-1, -1, -1$
- (c) $-1, 1 - 2\omega, 1 - 2\omega^2$
- (d) $-1, 1 + 2\omega, 1 + 2\omega^2$

59. If $|z^2 - 1| = |z|^2 + 1$, then z lies on [2004]

- (a) an ellipse (b) the imaginary axis
- (c) a circle (d) the real axis

60. The locus of the centre of a circle which touches the circle $|z - z_1| = a$ and $|z - z_2| = b$ externally (z, z_1 & z_2 are complex numbers) will be [2002]

- (a) an ellipse (b) a hyperbola
- (c) a circle (d) none of these

TOPIC 3

Solutions of Quadratic Equations, Sum and Product of Roots, Nature of Roots, Relation Between Roots and Co-efficients, Formation of an Equation with Given Roots.



61. If α and β be two roots of the equation $x^2 - 64x + 256 = 0$.

Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{\frac{1}{8}} + \left(\frac{\beta^3}{\alpha^5}\right)^{\frac{1}{8}}$ is: [Sep. 06, 2020 (I)]

- (a) 2 (b) 3 (c) 1 (d) 4

62. If α and β are the roots of the equation $2x(2x + 1) = 1$, then β is equal to: [Sep. 06, 2020 (II)]

- (a) $2\alpha(\alpha + 1)$ (b) $-2\alpha(\alpha + 1)$
- (c) $2\alpha(\alpha - 1)$ (d) $2\alpha^2$

63. The product of the roots of the equation $9x^2 - 18|x| + 5 = 0$, is: [Sep. 05, 2020 (I)]

- (a) $\frac{5}{9}$ (b) $\frac{25}{81}$ (c) $\frac{5}{27}$ (d) $\frac{25}{9}$

64. If α and β are the roots of the equation, $7x^2 - 3x - 2 = 0$,

the value of $\frac{\alpha}{1 - \alpha^2} + \frac{\beta}{1 - \beta^2}$ is equal to :

[Sep. 05, 2020 (II)]

- (a) $\frac{27}{32}$ (b) $\frac{1}{24}$ (c) $\frac{3}{8}$ (d) $\frac{27}{16}$

65. Let $u = \frac{2z+i}{z-ki}$, $z = x+iy$ and $k > 0$. If the curve represented by $\text{Re}(u) + \text{Im}(u) = 1$ intersects the y-axis at the points P and Q where $PQ = 5$, then the value of k is :

[Sep. 04, 2020 (I)]

- (a) $3/2$ (b) $1/2$ (c) 4 (d) 2

66. Let $\lambda \neq 0$ be in \mathbf{R} . If α and β are roots of the equation, $x^2 - x + 2\lambda = 0$ and α and γ are the roots of the equation,

$3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to :

[Sep. 04, 2020 (II)]

- (a) 27 (b) 18 (c) 9 (d) 36

67. If α and β are the roots of the equation $x^2 + px + 2 = 0$ and

$\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the equation $2x^2 + 2qx + 1 = 0$,

then $\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$ is equal to :

[Sep. 03, 2020 (I)]

- (a) $\frac{9}{4}(9+q^2)$ (b) $\frac{9}{4}(9-q^2)$
- (c) $\frac{9}{4}(9+p^2)$ (d) $\frac{9}{4}(9-p^2)$
68. The set of all real values of λ for which the quadratic equations, $(\lambda^2+1)x^2-4\lambda x+2=0$ always have exactly one root in the interval $(0, 1)$ is : [Sep. 03, 2020 (II)]
 (a) $(0, 2)$ (b) $(2, 4)$ (c) $(1, 3)$ (d) $(-3, -1)$
69. Let α and β be the roots of the equation, $5x^2+6x-2=0$. If $S_n = \alpha^n + \beta^n$, $n=1, 2, 3, \dots$, then : [Sep. 02, 2020 (I)]
 (a) $6S_6+5S_5=2S_4$ (b) $6S_6+5S_5+2S_4=0$
 (c) $5S_6+6S_5=2S_4$ (d) $5S_6+6S_5+2S_4=0$
70. The number of real roots of the equation, $e^{4x}+e^{3x}-4e^{2x}+e^x+1=0$ is: [Jan. 9, 2020 (I)]
 (a) 1 (b) 3 (c) 2 (d) 4
71. The least positive value of 'a' for which the equation, $2x^2+(a-10)x+\frac{33}{2}=2a$ has real roots is _____. [Jan. 8, 2020 (I)]
72. If the equation, $x^2+bx+45=0$ ($b \in R$) has conjugate complex roots and they satisfy $|z+1|=2\sqrt{10}$, then: [Jan. 8, 2020 (I)]
 (a) $b^2-b=30$ (b) $b^2+b=72$
 (c) $b^2-b=42$ (d) $b^2+b=12$
73. Let α and β be the roots of the equation $x^2-x-1=0$. If $p_k = (\alpha)^k + (\beta)^k$, $k \geq 1$, then which one of the following statements is not true? [Jan. 7, 2020 (II)]
 (a) $p_3 = p_5 - p_4$
 (b) $p_5 = 11$
 (c) $(p_1 + p_2 + p_3 + p_4 + p_5) = 26$
 (d) $p_5 = p_2 \cdot p_3$
74. Let α and β be two real roots of the equation $(k+1)\tan^2 x - \sqrt{2} \cdot \lambda \tan x = (1-k)$, where $k(\neq -1)$ and λ are real numbers. If $\tan^2(\alpha + \beta) = 50$, then a value of λ is: [Jan. 7, 2020 (I)]
 (a) $10\sqrt{2}$ (b) 10 (c) 5 (d) $5\sqrt{2}$
75. If α and β are the roots of the quadratic equation, $x^2+x\sin\theta-2\sin\theta=0$, $\theta \in \left(0, \frac{\pi}{2}\right)$, then $\frac{\alpha^{12}+\beta^{12}}{(\alpha^{-12}+\beta^{-12})(\alpha-\beta)^{24}}$ is equal to: [April 10, 2019 (I)]
 (a) $\frac{2^{12}}{(\sin\theta-4)^{12}}$ (b) $\frac{2^{12}}{(\sin\theta+8)^{12}}$
 (c) $\frac{2^{12}}{(\sin\theta-8)^6}$ (d) $\frac{2^6}{(\sin\theta+8)^{12}}$
76. The number of real roots of the equation $5+|2^x-1|=2^x(2^x-2)$ is: [April 10, 2019 (II)]
 (a) 3 (b) 2 (c) 4 (d) 1
77. Let $p, q \in R$. If $2-\sqrt{3}$ is a root of the quadratic equation, $x^2+px+q=0$, then: [April 9, 2019 (I)]
 (a) $p^2-4q+12=0$ (b) $q^2-4p-16=0$
 (c) $q^2+4p+14=0$ (d) $p^2-4q-12=0$
78. If m is chosen in the quadratic equation $(m^2+1)x^2-3x+(m^2+1)^2=0$ such that the sum of its roots is greatest, then the absolute difference of the cubes of its roots is: [April 09, 2019 (II)]
 (a) $10\sqrt{5}$ (b) $8\sqrt{3}$ (c) $8\sqrt{5}$ (d) $4\sqrt{3}$
79. The sum of the solutions of the equation $|\sqrt{x}-2|+\sqrt{x}(\sqrt{x}-4)+2=0$, ($x>0$) is equal to: [April 8, 2019 (I)]
 (a) 9 (b) 12 (c) 4 (d) 10
80. If α and β be the roots of the equation $x^2-2x+2=0$, then the least value of n for which $\left(\frac{\alpha}{\beta}\right)^n = 1$ is : [April 8, 2019 (I)]
 (a) 2 (b) 5 (c) 4 (d) 3
81. If λ be the ratio of the roots of the quadratic equation in x , $3m^2x^2+m(m-4)x+2=0$, then the least value of m for which $\lambda+\frac{1}{\lambda}=1$, is : [Jan. 12, 2019 (I)]
 (a) $2-\sqrt{3}$ (b) $4-3\sqrt{2}$
 (c) $-2+\sqrt{2}$ (d) $4-2\sqrt{3}$
82. If one real root of the quadratic equation $81x^2+kx+256=0$ is cube of the other root, then a value of k is : [Jan. 11, 2019 (I)]
 (a) -81 (b) 100 (c) 144 (d) -300
83. Consider the quadratic equation $(c-5)x^2-2cx+(c-4)=0$, $c \neq 5$. Let S be the set of all integral values of c for which one root of the equation lies in the interval $(0, 2)$ and its other root lies in the interval $(2, 3)$. Then the number of elements in S is: [Jan. 10, 2019 (I)]
 (a) 18 (b) 12 (c) 10 (d) 11
84. The value of λ such that sum of the squares of the roots of the quadratic equation, $x^2+(3-\lambda)x+2=\lambda$ has the least value is: [Jan. 10, 2019 (II)]
 (a) $\frac{15}{8}$ (b) 1 (c) $\frac{4}{9}$ (d) 2
85. Let α and β be two roots of the equation $x^2+2x+2=0$, then $\alpha^{15}+\beta^{15}$ is equal to: [Jan. 9, 2019 (I)]
 (a) -256 (b) 512 (c) -512 (d) 256
86. The number of all possible positive integral values of α for which the roots of the quadratic equation, $6x^2-11x+\alpha=0$ are rational numbers is: [Jan. 09, 2019 (II)]
 (a) 3 (b) 2 (c) 4 (d) 5



87. If both the roots of the quadratic equation $x^2 - mx + 4 = 0$ are real and distinct and they lie in the interval $[1, 5]$, then m lies in the interval: **[Jan. 09, 2019 (II)]**
 (a) $(-5, -4)$ (b) $(4, 5)$
 (c) $(5, 6)$ (d) $(3, 4)$
88. Let z_0 be a root of the quadratic equation, $x^2 + x + 1 = 0$. If $z = 3 + 6i z_0^{81} - 3i z_0^{93}$, then $\arg z$ is equal to: **[Jan. 09, 2019 (II)]**
 (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) 0
89. Let p, q and r be real numbers ($p \neq q, r \neq 0$), such that the roots of the equation $\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$ are equal in magnitude but opposite in sign, then the sum of squares of these roots is equal to. **[Online April 16, 2018]**
 (a) $p^2 + q^2 + r^2$ (b) $p^2 + q^2$
 (c) $2(p^2 + q^2)$ (d) $\frac{p^2 + q^2}{2}$
90. If an angle A of a ΔABC satisfies $5 \cos A + 3 = 0$, then the roots of the quadratic equation, $9x^2 + 27x + 20 = 0$ are. **[Online April 16, 2018]**
 (a) $\sin A, \sec A$ (b) $\sec A, \tan A$
 (c) $\tan A, \cos A$ (d) $\sec A, \cot A$
91. If $\tan A$ and $\tan B$ are the roots of the quadratic equation, $3x^2 - 10x - 25 = 0$ then the value of $3 \sin^2(A+B) - 10 \sin(A+B) \cdot \cos(A+B) - 25 \cos^2(A+B)$ is **[Online April 15, 2018]**
 (a) 25 (b) -25 (c) -10 (d) 10
92. If $f(x)$ is a quadratic expression such that $f(a) + f(b) = 0$, and -1 is a root of $f(x) = 0$, then the other root of $f(x) = 0$ is **[Online April 15, 2018]**
 (a) $-\frac{5}{8}$ (b) $-\frac{8}{5}$ (c) $\frac{5}{8}$ (d) $\frac{8}{5}$
93. If $\alpha, \beta \in \mathbb{C}$ are the distinct roots, of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to: **[2018]**
 (a) 0 (b) 1 (c) 2 (d) -1
94. If, for a positive integer n , the quadratic equation, $x(x+1) + (x+1)(x+2) + \dots + (x+n-1)(x+n) = 10n$ has two consecutive integral solutions, then n is equal to: **[2017]**
 (a) 11 (b) 12 (c) 9 (d) 10
95. The sum of all the real values of x satisfying the equation $2^{(x-1)}(x^2+5x-50) = 1$ is: **[Online April 9, 2017]**
 (a) 16 (b) 14 (c) -4 (d) -5
96. Let $p(x)$ be a quadratic polynomial such that $p(0)=1$. If $p(x)$ leaves remainder 4 when divided by $x-1$ and it leaves remainder 6 when divided by $x+1$; then : **[Online April 8, 2017]**
 (a) $p(b) = 11$ (b) $p(b) = 19$
 (c) $p(-2) = 19$ (d) $p(-2) = 11$
97. The sum of all real values of x satisfying the equation $(x^2 - 5x + 5)^{x^2 + 4x - 60} = 1$ is : **[2016]**
 (a) 6 (b) 5 (c) 3 (d) -4
98. If x is a solution of the equation, $\sqrt{2x+1} - \sqrt{2x-1} = 1, \left(x \geq \frac{1}{2}\right)$, then $\sqrt{4x^2 - 1}$ is equal to : **[Online April 10, 2016]**
 (a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $2\sqrt{2}$ (d) 2
99. Let α and β be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is equal to: **[2015]**
 (a) 3 (b) -3 (c) 6 (d) -6
100. If the two roots of the equation, $(a-1)(x^4 + x^2 + 1) + (a+1)(x^2 + x + 1)^2 = 0$ are real and distinct, then the set of all values of 'a' is : **[Online April 11, 2015]**
 (a) $\left(0, \frac{1}{2}\right)$ (b) $\left(-\frac{1}{2}, 0\right) \cup \left(0, \frac{1}{2}\right)$
 (c) $\left(-\frac{1}{2}, 0\right)$ (d) $(-\infty, -2) \cup (2, \infty)$
101. If $2 + 3i$ is one of the roots of the equation $2x^3 - 9x^2 + kx - 13 = 0, k \in \mathbb{R}$, then the real root of this equation : **[Online April 10, 2015]**
 (a) exists and is equal to $-\frac{1}{2}$.
 (b) exists and is equal to $\frac{1}{2}$.
 (c) exists and is equal to 1.
 (d) does not exist.
102. If $a \in \mathbb{R}$ and the equation $-3(x - [x])^2 + 2(x - [x]) + a^2 = 0$ (where $[x]$ denotes the greatest integer $\leq x$) has no integral solution, then all possible values of a lie in the interval: **[2014]**
 (a) $(-2, -1)$ (b) $(-\infty, -2) \cup (2, \infty)$
 (c) $(-1, 0) \cup (0, 1)$ (d) $(1, 2)$
103. The equation $\sqrt{3x^2 + x + 5} = x - 3$, where x is real, has; **[Online April 19, 2014]**
 (a) no solution (b) exactly one solution
 (c) exactly two solution (d) exactly four solution

104. The sum of the roots of the equation, $x^2 + |2x - 3| - 4 = 0$, is: **[Online April 12, 2014]**
 (a) 2 (b) -2 (c) $\sqrt{2}$ (d) $-\sqrt{2}$
105. If α and β are roots of the equation, $x^2 - 4\sqrt{2}kx + 2e^{4lnk} - 1 = 0$ for some k , and $\alpha^2 + \beta^2 = 66$, then $\alpha^3 + \beta^3$ is equal to: **[Online April 11, 2014]**
 (a) $248\sqrt{2}$ (b) $280\sqrt{2}$ (c) $-32\sqrt{2}$ (d) $-280\sqrt{2}$
106. If $\frac{1}{\sqrt{\alpha}}$ and $\frac{1}{\sqrt{\beta}}$ are the roots of the equation, $ax^2 + bx + 1 = 0$ ($a \neq 0$, $a, b, \in \mathbb{R}$), then the equation, $x(x + b^3) + (a^3 - 3abx) = 0$ as roots : **[Online April 9, 2014]**
 (a) $\alpha^{3/2}$ and $\beta^{3/2}$ (b) $\alpha\beta^{1/2}$ and $\alpha^{1/2}\beta$
 (c) $\sqrt{\alpha\beta}$ and $\alpha\beta$ (d) $\alpha\frac{3}{2}$ and $\beta\frac{3}{2}$
107. If p and q are non-zero real numbers and $\alpha^3 + \beta^3 = -p$, $\alpha\beta = q$, then a quadratic equation whose roots are $\frac{\alpha^2}{\beta}, \frac{\beta^2}{\alpha}$ is : **[Online April 25, 2013]**
 (a) $px^2 - qx + p^2 = 0$ (b) $qx^2 + px + q^2 = 0$
 (c) $px^2 + qx + p^2 = 0$ (d) $qx^2 - px + q^2 = 0$
108. If α and β are roots of the equation $x^2 + px + \frac{3p}{4} = 0$, such that $|\alpha - \beta| = \sqrt{10}$, then p belongs to the set : **[Online April 22, 2013]**
 (a) $\{2, -5\}$ (b) $\{-3, 2\}$ (c) $\{-2, 5\}$ (d) $\{3, -5\}$
109. If a complex number z satisfies the equation $z + \sqrt{2}|z + 1| + i = 0$, then $|z|$ is equal to : **[Online April 22, 2013]**
 (a) 2 (b) $\sqrt{3}$ (c) $\sqrt{5}$ (d) 1
110. Let $p, q, r \in \mathbb{R}$ and $r > p > 0$. If the quadratic equation $px^2 + qx + r = 0$ has two complex roots α and β , then $|\alpha| + |\beta|$ is **[Online May 19, 2012]**
 (a) equal to 1
 (b) less than 2 but not equal to 1
 (c) greater than 2
 (d) equal to 2
111. If the sum of the square of the roots of the equation $x^2 - (\sin\alpha - 2)x - (1 + \sin\alpha) = 0$ is least, then α is equal to **[Online May 12, 2012]**
 (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$
112. The value of k for which the equation $(k - 2)x^2 + 8x + k + 4 = 0$ has both roots real, distinct and negative is **[Online May 7, 2012]**
 (a) 6 (b) 3 (c) 4 (d) 1
113. Let for $a \neq a_1 \neq 0$,
 $f(x) = ax^2 + bx + c$, $g(x) = a_1x^2 + b_1x + c_1$
 and $p(x) = f(x) - g(x)$.
 If $p(x) = 0$ only for $x = -1$ and $p(-2) = 2$, then the value of $p(b)$ is : **[2011 RS]**
 (a) 3 (b) 9 (c) 6 (d) 18
114. Sachin and Rahul attempted to solve a quadratic equation. Sachin made a mistake in writing down the constant term and ended up in roots (4,3). Rahul made a mistake in writing down coefficient of x to get roots (3,2). The correct roots of equation are : **[2011 RS]**
 (a) 6, 1 (b) 4, 3 (c) -6, -1 (d) -4, -3
115. Let α, β be real and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line $\text{Re } z = 1$, then it is necessary that : **[2011]**
 (a) $\beta \in (-1, 0)$ (b) $|\beta| = 1$
 (c) $\beta \in (1, \infty)$ (d) $\beta \in (0, 1)$
116. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$ **[2010]**
 (a) -1 (b) 1 (c) 2 (d) -2
117. If the roots of the equation $bx^2 + cx + a = 0$ be imaginary, then for all real values of x , the expression $3b^2x^2 + 6bcx + 2c^2$ is : **[2009]**
 (a) less than $4ab$ (b) greater than $-4ab$
 (c) less than $-4ab$ (d) greater than $4ab$
118. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is **[2007]**
 (a) $(3, \infty)$ (b) $(-\infty, -3)$ (c) $(-3, 3)$ (d) $(-3, \infty)$
119. All the values of m for which both roots of the equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than -2 but less than 4, lie in the interval **[2006]**
 (a) $-2 < m < 0$ (b) $m > 3$
 (c) $-1 < m < 3$ (d) $1 < m < 4$
120. If the roots of the quadratic equation $x^2 + px + q = 0$ are $\tan 30^\circ$ and $\tan 15^\circ$, respectively, then the value of $2 + q - p$ is **[2006]**
 (a) 2 (b) 3 (c) 0 (d) 1
121. If $z^2 + z + 1 = 0$, where z is complex number, then the value of $\left(z + \frac{1}{z}\right)^2 + \left(z^2 + \frac{1}{z^2}\right)^2 + \left(z^3 + \frac{1}{z^3}\right)^2 + \dots + \left(z^6 + \frac{1}{z^6}\right)^2$ is **[2006]**
 (a) 18 (b) 54 (c) 6 (d) 12



122. In a triangle PQR , $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and

$-\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0$, $a \neq 0$ then

[2005]

- (a) $a = b + c$ (b) $c = a + b$
 (c) $b = c$ (d) $b = a + c$

123. If the roots of the equation $x^2 - bx + c = 0$ be two consecutive integers, then $b^2 - 4c$ equals [2005]

- (a) -2 (b) 3 (c) 2 (d) 1

124. If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of 'q' is [2004]

- (a) 4 (b) 12 (c) 3 (d) $\frac{49}{4}$

125. If $(1-p)$ is a root of quadratic equation $x^2 + px + (1-p) = 0$ then its root are [2004]

- (a) -1, 2 (b) -1, 1 (c) 0, -1 (d) 0, 1

126. The number of real solutions of the equation

$x^2 - 3|x| + 2 = 0$ is [2003]

- (a) 3 (b) 2 (c) 4 (d) 1

127. The value of 'a' for which one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other is [2003]

- (a) $-\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $-\frac{2}{3}$ (d) $\frac{1}{3}$

128. Let Z_1 and Z_2 be two roots of the equation

$Z^2 + aZ + b = 0$, Z being complex. Further, assume that the origin, Z_1 and Z_2 form an equilateral triangle. Then

[2003]

- (a) $a^2 = 4b$ (b) $a^2 = b$
 (c) $a^2 = 2b$ (d) $a^2 = 3b$

129. If p and q are the roots of the equation $x^2 + px + q = 0$, then [2002]

- (a) $p = 1, q = -2$ (b) $p = 0, q = 1$
 (c) $p = -2, q = 0$ (d) $p = -2, q = 1$

130. Product of real roots of the equation $t^2 x^2 + |x| + 9 = 0$ [2002]

- (a) is always positive (b) is always negative
 (c) does not exist (d) none of these

131. Difference between the corresponding roots of $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ is same and $a \neq b$, then [2002]

- (a) $a + b + 4 = 0$ (b) $a + b - 4 = 0$
 (c) $a - b - 4 = 0$ (d) $a - b + 4 = 0$

132. If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$ then the equation having α/β and β/α as its roots is [2002]

- (a) $3x^2 - 19x + 3 = 0$ (b) $3x^2 + 19x - 3 = 0$
 (c) $3x^2 - 19x - 3 = 0$ (d) $x^2 - 5x + 3 = 0$.

TOPIC 4

Condition for Common Roots, Maximum and Minimum value of Quadratic Equation, Quadratic Expression in two Variables, Solution of Quadratic Inequalities.



133. If 5, 5r, 5r² are the lengths of the sides of a triangle, then r cannot be equal to: [Jan. 10, 2019 (I)]

- (a) $\frac{3}{4}$ (b) $\frac{5}{4}$ (c) $\frac{7}{4}$ (d) $\frac{3}{2}$

134. Let $a, b \in \mathbb{R}$, $a \neq 0$ be such that the equation, $ax^2 - 2bx + 5 = 0$ has a repeated root α , which is also a root of the equation, $x^2 - 2bx - 10 = 0$. If β is the other root of this equation, then $\alpha^2 + \beta^2$ is equal to : [Jan. 9, 2020 (II)]

- (a) 25 (b) 26 (c) 28 (d) 24

135. If $\lambda \in \mathbb{R}$ is such that the sum of the cubes of the roots of the equation, $x^2 + (2 - \lambda)x + (10 - \lambda) = 0$ is minimum, then the magnitude of the difference of the roots of this equation is [Online April 15, 2018]

- (a) 20 (b) $2\sqrt{5}$ (c) $2\sqrt{7}$ (d) $4\sqrt{2}$

136. If $|z - 3 + 2i| \leq 4$ then the difference between the greatest value and the least value of $|z|$ is [Online April 15, 2018]

- (a) $\sqrt{13}$ (b) $2\sqrt{13}$ (c) 8 (d) $4 + \sqrt{13}$

137. If the equations $x^2 + bx - 1 = 0$ and $x^2 + x + b = 0$ have a common root different from -1, then $|b|$ is equal to : [Online April 9, 2016]

- (a) 2 (b) 3 (c) $\sqrt{3}$ (d) $\sqrt{2}$

138. If non-zero real numbers b and c are such that $\min f(x) > \max g(x)$, where $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$ ($x \in \mathbb{R}$);

then $\left|\frac{c}{b}\right|$ lies in the interval: [Online April 19, 2014]

- (a) $\left(0, \frac{1}{2}\right)$ (b) $\left[\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$

- (c) $\left[\frac{1}{\sqrt{2}}, \sqrt{2}\right]$ (d) $(\sqrt{2}, \infty)$

139. If equations $ax^2 + bx + c = 0$ ($a, b, c \in \mathbb{R}, a \neq 0$) and $2x^2 + 3x + 4 = 0$ have a common root, then $a : b : c$ equals:
[Online April 9, 2014]
(a) 1 : 2 : 3 (b) 2 : 3 : 4 (c) 4 : 3 : 2 (d) 3 : 2 : 1
140. If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, $a, b, c \in \mathbb{R}$, have a common root, then $a : b : c$ is [2013]
(a) 1 : 2 : 3 (b) 3 : 2 : 1 (c) 1 : 3 : 2 (d) 3 : 1 : 2
141. The least integral value α of x such that $\frac{x-5}{x^2+5x-14} > 0$, satisfies :
[Online April 23, 2013]
(a) $\alpha^2 + 3\alpha - 4 = 0$ (b) $\alpha^2 - 5\alpha + 4 = 0$
(c) $\alpha^2 - 7\alpha + 6 = 0$ (d) $\alpha^2 + 5\alpha - 6 = 0$
142. The values of 'a' for which one root of the equation $x^2 - (a+1)x + a^2 + a - 8 = 0$ exceeds 2 and the other is lesser than 2, are given by : [Online April 9, 2013]
(a) $3 < a < 10$ (b) $a \geq 10$
(c) $-2 < a < 3$ (d) $a \leq -2$
143. If $\left| z - \frac{4}{z} \right| = 2$, then the maximum value of $|z|$ is equal to :
[2009]
(a) $\sqrt{5} + 1$ (b) 2 (c) $2 + \sqrt{2}$ (d) $\sqrt{3} + 1$
144. The quadratic equations $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is [2009]
(a) 1 (b) 4 (c) 3 (d) 2
145. If x is real, the maximum value of $\frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$ is [2006]
(a) $\frac{1}{4}$ (b) 41 (c) 1 (d) $\frac{17}{7}$
146. If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval [2005]
(a) (5, 6] (b) (6, ∞) (c) ($-\infty$, 4) (d) [4, 5]
147. The value of a for which the sum of the squares of the roots of the equation $x^2 - (a-2)x - a - 1 = 0$ assume the least value is [2005]
(a) 1 (b) 0 (c) 3 (d) 2



Hints & Solutions



1. (b) Let $z = \frac{3+i\sin\theta}{4-i\cos\theta}$, after rationalising

$$z = \frac{(3+i\sin\theta)}{(4-i\cos\theta)} \times \frac{(4+i\cos\theta)}{(4+i\cos\theta)}$$

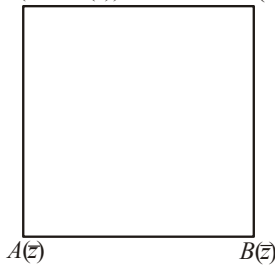
As z is purely real

$$\Rightarrow 3\cos\theta + 4\sin\theta = 0 \Rightarrow \tan\theta = -\frac{3}{4}$$

$$\arg(\sin\theta + i\cos\theta) = \pi + \tan^{-1}\left(\frac{\cos\theta}{\sin\theta}\right)$$

$$= \pi + \tan^{-1}\left(-\frac{4}{3}\right) = \pi - \tan^{-1}\left(\frac{4}{3}\right)$$

2. (c) $D(z-2\operatorname{Re}(z))$ $C(\bar{z}-2\operatorname{Re}(\bar{z}))$



Let $z = x + iy$

\therefore Length of side of square = 4 units

Then, $|z - \bar{z}| = 4 \Rightarrow |2iy| = 4 \Rightarrow |y| = 2$

Also, $|z - (z - 2\operatorname{Re}(z))| = 4$

$\Rightarrow |2\operatorname{Re}(z)| = 4 \Rightarrow |2x| = 4 \Rightarrow |x| = 2$

$$\therefore |z| = \sqrt{x^2 + y^2} = \sqrt{4+4} = 2\sqrt{2}$$

3. (c) $\therefore -1 + \sqrt{3}i = 2 \cdot e^{\frac{2\pi i}{3}}$ and $1 - i = \sqrt{2} \cdot e^{\frac{i\pi}{4}}$

$$\therefore \left(\frac{-1 + \sqrt{3}i}{1 - i}\right)^{30} = \left(\sqrt{2}e^{\left(\frac{2\pi}{3} + \frac{\pi}{4}\right)i}\right)^{30}$$

$$= 2^{15} \cdot e^{\frac{\pi i}{2}} = -2^{15} \cdot i$$

4. (4)

Given that $\left(\frac{1+i}{1-i}\right)^{m/2} = \left(\frac{1+i}{i-1}\right)^{n/3} = 1$

$$\Rightarrow \left(\frac{(1+i)^2}{2}\right)^{m/2} = \left(\frac{(1+i)^2}{-2}\right)^{n/3} = 1$$

$$\Rightarrow i^{m/2} = (-i)^{n/3} = 1$$

m (least) = 8, n (least) = 12

GCD(8, 12) = 4.

5. (b) Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

$$\therefore |z_1 - 1| = \operatorname{Re}(z_1)$$

$$\Rightarrow (x_1 - 1)^2 + y_1^2 = x_1^2$$

$$\Rightarrow y_1^2 - 2x_1 + 1 = 0 \quad \dots(i)$$

$$|z_2 - 1| = \operatorname{Re}(z_2) \Rightarrow (x_2 - 1)^2 + y_2^2 = x_2^2$$

$$\Rightarrow y_2^2 - 2x_2 + 1 = 0 \quad \dots(ii)$$

From eqn. (i) - (ii),

$$y_1^2 - y_2^2 - 2(x_1 - x_2) = 0$$

$$\Rightarrow y_1 + y_2 = 2\left(\frac{x_1 - x_2}{y_1 - y_2}\right) \quad \dots(iii)$$

$$\therefore \arg(z_1 - z_2) = \frac{\pi}{6}$$

$$\Rightarrow \tan^{-1}\left(\frac{y_1 - y_2}{x_1 - x_2}\right) = \frac{\pi}{6}$$

$$\Rightarrow \frac{y_1 - y_2}{x_1 - x_2} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{2}{y_1 + y_2} = \frac{1}{\sqrt{3}} \quad \left[\text{From, } \frac{y_1 - y_2}{x_1 - x_2} = \frac{2}{y_1 + y_2} \right]$$

$$\therefore y_1 + y_2 = 2\sqrt{3} \Rightarrow \operatorname{Im}(z_1 + z_2) = 2\sqrt{3}$$

6. (b) Let $z = x + iy$

Then, $\left|\frac{z-i}{z+2i}\right| = 1 \Rightarrow x^2 + (y-1)^2$

$$= x^2 + (y+2)^2 \Rightarrow -2y + 1 = 4y + 4$$

$$\Rightarrow 6y = -3 \Rightarrow y = -\frac{1}{2}$$

$$\therefore |z| = \frac{5}{2} \Rightarrow x^2 + y^2 = \frac{25}{4}$$

$$\Rightarrow x^2 = \frac{24}{4} = 6$$

$$\therefore z = x + iy \Rightarrow z = \pm \sqrt{6} - \frac{i}{2}$$

$$|z + 3i| = \sqrt{6 + \frac{25}{4}} = \sqrt{\frac{49}{4}}$$

$$\Rightarrow |z + 3i| = \frac{7}{2}$$

7. (c) $z = x + iy$
 $|x| + |y| = 4$

$$|z| = \sqrt{x^2 + y^2}$$

Minimum value of

$$|z| = 2\sqrt{2}$$

Maximum value of

$$|z| = 4$$

$$|z| \in [\sqrt{8}, \sqrt{16}]$$

So, $|z|$ can't be $\sqrt{7}$.

8. (c) Let $\text{Re}(z) = x$ i.e., $z = x + 10i$

$$2z - n = (2i - 1)(2z + n)$$

$$(2x - n) + 20i = (2i - 1)((2x + n) + 20i)$$

On comparing real and imaginary parts,

$$-(2x + n) - 40 = 2x - n \text{ and } 20 = 4x + 2n - 20$$

$$\Rightarrow 4x = -40 \text{ and } 40 = -40 + 2n$$

$$\Rightarrow x = -10 \text{ and } n = 40$$

$$\text{Hence, } \text{Re}(z) = -10$$

9. (b) Given equation is, $|z - 1| = |z - i|$

$$\Rightarrow (x - 1)^2 + y^2 = x^2 + (y - 1)^2 \quad [\text{Here, } z = x + iy]$$

$$\Rightarrow 1 - 2x = 1 - 2y \Rightarrow x - y = 0$$

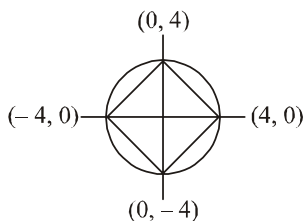
Hence, locus is straight line with slope 1.

10. (a) $z = \frac{(1+i)^2}{a-i} \times \frac{a+i}{a+i}$

$$z = \frac{(1-1+2i)(a+i)}{a^2+1} = \frac{2ai-2}{a^2+1}$$

$$|z| = \sqrt{\left(\frac{-2}{a^2+1}\right)^2 + \left(\frac{2a}{a^2+1}\right)^2} = \sqrt{\frac{4+4a^2}{(a^2+1)^2}}$$

$$\Rightarrow |z| = \sqrt{\frac{4(1+a^2)}{(1+a^2)^2}} = \frac{2}{\sqrt{1+a^2}} \quad \dots(i)$$



Since, it is given that $|z| = \sqrt{\frac{2}{5}}$

Then, from equation (i),

$$\sqrt{\frac{2}{5}} = \frac{2}{\sqrt{1+a^2}}$$

Now, square on both side; we get

$$\Rightarrow 1 + a^2 = 10 \Rightarrow a = \pm 3$$

Since, it is given that $a > 0 \Rightarrow a = 3$

$$\text{Then, } z = \frac{(1+i)^2}{a-i} = \frac{1+i^2+2i}{3-i} = \frac{2i}{3-i}$$

$$= \frac{2i(3+i)}{10} = \frac{-1+3i}{5}$$

$$\text{Hence, } \bar{z} = \frac{-1}{5} - \frac{3}{5}i$$

11. (c) Given $|z\omega| = 1 \quad \dots(i)$

$$\text{and } \arg\left(\frac{z}{\omega}\right) = \frac{\pi}{2} \quad \dots(ii)$$

$$\therefore \frac{z}{\omega} + \frac{\bar{z}}{\omega} = 0 \quad \left[\because \text{Re}\left(\frac{z}{\omega}\right) = 0 \right]$$

$$\Rightarrow z\bar{\omega} = -\bar{z}\omega$$

from equation (i), $z\bar{z}\omega\bar{\omega} = 1$ [using $z\bar{z} = |z|^2$]

$$(\bar{z}\omega)^2 = -1 \Rightarrow \bar{z}\omega = \pm i$$

$$\text{from equation (ii), } -\arg(\bar{z}) - \arg\omega = \frac{\pi}{2} - \arg(\bar{z}\omega) = \frac{-\pi}{2}$$

$$\text{Hence, } \bar{z}\omega = -i$$

12. (c) $\omega = \frac{5+3z}{5-5z} \Rightarrow 5\omega - 5\omega z = 5 + 3z$

$$\Rightarrow 5\omega - 5 = z(3+5\omega) \Rightarrow z = \frac{5(\omega-1)}{3+5\omega}$$

$$\because |z| < 1, \therefore 5|\omega-1| < |3+5\omega|$$

$$\Rightarrow 25(\omega\bar{\omega} - \omega - \bar{\omega} + 1) < 9 + 25\omega\bar{\omega} + 15\omega + 15\bar{\omega}$$

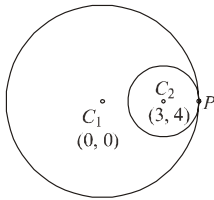
$$(\because |z|^2 = z\bar{z})$$

$$\Rightarrow 16 < 40\omega + 40\bar{\omega} \Rightarrow \omega + \bar{\omega} > \frac{2}{5} \Rightarrow 2\text{Re}(\omega) > \frac{2}{5}$$

$$\Rightarrow \text{Re}(\omega) > \frac{1}{5}$$

13. (a) Let $t = \frac{z - \alpha}{z + \alpha}$
 $\therefore t$ is purely imaginary number.
 $\therefore t + \bar{t} = 0$
 $\Rightarrow \frac{z - \alpha}{z + \alpha} + \frac{\bar{z} - \alpha}{\bar{z} + \alpha} = 0$
 $\Rightarrow (z - \alpha)(\bar{z} + \alpha) + (\bar{z} - \alpha)(z + \alpha) = 0$
 $\Rightarrow z\bar{z} - \alpha^2 + z\bar{z} - \alpha^2 = 0$
 $\Rightarrow z\bar{z} - \alpha^2 = 0$
 $\Rightarrow |z|^2 - \alpha^2 = 0$
 $\Rightarrow \alpha^2 = 4$
 $\Rightarrow \alpha = \pm 2$

14. (a) $|z_1| = 9, |z_2 - 3 - 4i| = 4$
 z_1 lies on a circle with centre $C_1(0, 0)$ and radius $r_1 = 9$
 z_2 lies on a circle with centre $C_2(3, 4)$ and radius $r_2 = 4$
 So, minimum value of $|z_1 - z_2|$ is zero at point of contact (i.e. A)



15. (b) Since, $|z| + z = 3 + i$
 Let $z = a + ib$, then
 $|z| + z = 3 + i \Rightarrow \sqrt{a^2 + b^2} + a + ib = 3 + i$
 Compare real and imaginary coefficients on both sides
 $b = 1, \sqrt{a^2 + b^2} + a = 3$
 $\sqrt{a^2 + 1} = 3 - a$
 $a^2 + 1 = a^2 + 9 - 6a$
 $6a = 8 \Rightarrow a = \frac{4}{3}$
 Then,
 $|z| = \sqrt{\left(\frac{4}{3}\right)^2 + 1} = \sqrt{\frac{16}{9} + 1} = \frac{5}{3}$

16. (none) Let $z_1 = r_1 e^{i\theta}$ and $z_2 = r_2 e^{i\phi}$
 $3|z_1| = 4|z_2| \Rightarrow 3r_1 = 4r_2$

$$z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1} = \frac{3r_1}{2r_2} e^{i(\theta - \phi)} + \frac{2}{3} \frac{r_2}{r_1} e^{i(\phi - \theta)}$$

$$= \frac{3}{2} \times \frac{4}{3} (\cos(\theta - \phi) + i \sin(\theta - \phi)) +$$

$$\frac{2}{3} \times \frac{3}{4} [\cos(\theta - \phi) - i \sin(\theta - \phi)]$$

$$z = \left(2 + \frac{1}{2}\right) \cos(\theta - \phi) + i \left(2 - \frac{1}{2}\right) \sin(\theta - \phi)$$

$$\therefore |z| = \sqrt{\frac{25}{4} \cos^2(\theta - \phi) + \frac{9}{4} \sin^2(\theta - \phi)}$$

$$= \sqrt{\frac{16}{4} \cos^2(\theta - \phi) + \frac{9}{4}} \Rightarrow \frac{3}{2} \leq |z| \leq \frac{5}{2}$$

17. (d) Suppose $z = \frac{3 + 2i \sin \theta}{1 - 2i \sin \theta}$
 Since, z is purely imaginary, then $z + \bar{z} = 0$
 $\Rightarrow \frac{3 + 2i \sin \theta}{1 - 2i \sin \theta} + \frac{3 - 2i \sin \theta}{1 + 2i \sin \theta} = 0$
 $\Rightarrow \frac{(3 + 2i \sin \theta)(1 + 2i \sin \theta) + (3 - 2i \sin \theta)(1 - 2i \sin \theta)}{1 + 4 \sin^2 \theta} = 0$
 $\Rightarrow \sin^2 \theta = \frac{3}{4} \Rightarrow \sin \theta = \frac{\sqrt{3}}{2}$
 $\Rightarrow \theta = -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$

Now, the sum of elements in $A = -\frac{\pi}{3} + \frac{\pi}{3} + \frac{2\pi}{3} = \frac{2\pi}{3}$

18. (a) $\because |z| = 1$ & $Re z \neq 1$
 Suppose $z = x + iy \Rightarrow x^2 + y^2 = 1$ (i)
 Now, $w = \frac{1 + (1 - 8\alpha)z}{1 - z}$
 $\Rightarrow w = \frac{1 + (1 - 8\alpha)(x + iy)}{1 - (x + iy)}$
 $\Rightarrow w = \frac{1 + (1 - 8\alpha)(x + iy)((1 - x) + iy)}{1 - (x + iy)((1 - x) + iy)}$
 $\Rightarrow w = \frac{[(1 + x(1 - 8\alpha))(1 - x) - (1 - 8\alpha)y^2]}{(1 - x)^2 + y^2} + i \frac{[(1 + x(1 - 8\alpha))y - (1 - 8\alpha)y(1 - x)]}{(1 - x)^2 + y^2}$

As, w is purely imaginary. So,

$$\operatorname{Re} w = \frac{[(1+x(1-8\alpha))(1-x) - (1-8\alpha)y^2]}{(1-x)^2 + y^2} = 0$$

$$\begin{aligned} \Rightarrow (1-x) + x(1-8\alpha)(1-x) &= (1-8\alpha)y^2 \\ \Rightarrow (1-x) + x(1-8\alpha) - x^2(1-8\alpha) &= (1-8\alpha)y^2 \\ \Rightarrow (1-x) + x(1-8\alpha) &= 1-8\alpha \quad [\text{From (i), } x^2 + y^2 = 1] \\ \Rightarrow 1-8\alpha &= 1 \\ \Rightarrow \alpha &= 0 \\ \therefore \alpha &\in \{0\} \end{aligned}$$

19. (b) Rationalizing the given expression

$$\frac{(2+3i\sin\theta)(1+2i\sin\theta)}{1+4\sin^2\theta}$$

For the given expression to be purely imaginary, real part of the above expression should be equal to zero.

$$\begin{aligned} \Rightarrow \frac{2-6\sin^2\theta}{1+4\sin^2\theta} &= 0 \quad \Rightarrow \sin^2\theta = \frac{1}{3} \\ \Rightarrow \sin\theta &= \pm \frac{1}{\sqrt{3}} \Rightarrow \theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right) \end{aligned}$$

20. (b) Let $z = re^{i\theta}$

$$\text{Consider } \frac{\operatorname{Im} z^5}{(\operatorname{Im} z)^5} = \frac{r^5(\sin 5\theta)}{r^5(\sin\theta)^5}$$

$$(\because e^{i\theta} = \cos\theta + i\sin\theta)$$

$$= \frac{\sin 5\theta}{\sin^5\theta} = \frac{16\sin^5\theta - 20\sin^3\theta + 5\sin\theta}{\sin^5\theta}$$

$$\begin{aligned} &= \frac{16\sin^5\theta}{\sin^5\theta} - \frac{20\sin^3\theta}{\sin^5\theta} + \frac{5\sin\theta}{\sin^5\theta} \\ &= 5 \operatorname{cosec}^4\theta - 20 \operatorname{cosec}^2\theta + 16 \end{aligned}$$

$$\text{minimum value of } \frac{\operatorname{Im} z^5}{(\operatorname{Im} z)^5} \text{ is } -4.$$

21. (d) We know minimum value of $|Z_1 + Z_2|$ is

$$||Z_1| - |Z_2||. \text{ Thus minimum value of } \left| Z + \frac{1}{2} \right| \text{ is } \left| |Z| - \frac{1}{2} \right|$$

$$\leq \left| Z + \frac{1}{2} \right| \leq |Z| + \frac{1}{2}$$

Since, $|Z| \geq 2$ therefore

$$2 - \frac{1}{2} < \left| Z + \frac{1}{2} \right| < 2 + \frac{1}{2}$$

$$\Rightarrow \frac{3}{2} < \left| Z + \frac{1}{2} \right| < \frac{5}{2}$$

22. (b) Let $z = 1 + i\alpha$, $\alpha \in \mathbb{R}$

$$z^2 = (1 + i\alpha)(1 + i\alpha)$$

$$x + iy = (1 + 2i\alpha - \alpha^2)$$

On comparing real and imaginary parts, we get

$$x = 1 - \alpha^2, y = 2\alpha$$

Now, consider option (b), which is

$$y^2 + 4x - 4 = 0$$

$$\text{LHS : } y^2 + 4x - 4 = (2\alpha)^2 + 4(1 - \alpha^2) - 4$$

$$= 4\alpha^2 + 4 - 4\alpha^2 - 4$$

$$= 0 = \text{R.H.S.}$$

$$\text{Hence, } y^2 + 4x - 4 = 0$$

23. (c) Let $z = x + iy$

$\frac{z-i}{z+i}$ is purely imaginary means its real part is zero.

$$\frac{x+iy-i}{x+iy+i} = \frac{x+i(y-1)}{x+i(y+1)} \times \frac{x-i(y+1)}{x-i(y+1)}$$

$$= \frac{x^2 - 2ix(y+1) + xi(y-1) + y^2 - 1}{x^2 + (y+1)^2}$$

$$= \frac{x^2 + y^2 - 1}{x^2 + (y+1)^2} - \frac{2xi}{x^2 + (y+1)^2}$$

for pure imaginary, we have

$$\frac{x^2 + y^2 - 1}{x^2 + (y+1)^2} = 0$$

$$\Rightarrow x^2 + y^2 = 1$$

$$\Rightarrow (x+iy)(x-iy) = 1$$

$$\Rightarrow x + iy = \frac{1}{x-iy} = z$$

$$\text{and } \frac{1}{z} = x - iy$$

$$z + \frac{1}{z} = (x+iy) + (x-iy) = 2x$$

$$\left(z + \frac{1}{z} \right) \text{ is any non-zero real number}$$

24. (a) Consider $\arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right)$

$$= \arg(z_1) - \arg(z_4) + \arg(z_2) - \arg(z_3)$$

$$= (\arg(z_1) + \arg(z_2)) - (\arg(z_3) + \arg(z_4))$$

$$\text{given } \begin{pmatrix} z_2 = \bar{z}_1 & \\ z_4 = \bar{z}_3 & \end{pmatrix}$$

$$= (\arg(z_1) + \arg(\bar{z}_1)) - (\arg(z_3) + \arg(\bar{z}_3))$$

$$\left\{ \begin{array}{l} \text{also } (\arg(\bar{z}_1) = -\arg(z_1)) \\ \arg(\bar{z}_3) = -\arg(z_3) \end{array} \right\}$$

$$= (\arg(z_1) - \arg(z_1)) - (\arg(z_3) - \arg(z_3))$$

$$= 0 - 0 = 0$$

25. (d) Consider the equation

$$w - \bar{w}z = k(1 - z), k \in R$$

Clearly $z \neq 1$ and $\frac{w - \bar{w}z}{1 - z}$ is purely real

$$\therefore \frac{\overline{w - \bar{w}z}}{1 - z} = \frac{w - \bar{w}z}{1 - z}$$

$$\Rightarrow \frac{\bar{w} - w\bar{z}}{1 - \bar{z}} = \frac{w - \bar{w}z}{1 - z}$$

$$\Rightarrow \bar{w} - \bar{w}z - w\bar{z} + w\bar{z}\bar{z} = w - w\bar{z} - \bar{w}z + \bar{w}z\bar{z}$$

$$\Rightarrow \bar{w} + w|z|^2 = w + \bar{w}|z|^2$$

$$\Rightarrow (w - \bar{w})(|z|^2) = w - \bar{w}$$

$$\Rightarrow |z|^2 = 1 \quad (\because \operatorname{Im} w \neq 0)$$

$$\Rightarrow |z| = 1 \text{ and } z \neq 1$$

\therefore The required set is $\{z : |z| = 1, z \neq 1\}$

26. (c) Given $|z| = 1, \arg z = \theta$

$$\Rightarrow \bar{z} = \frac{1}{z}$$

$$\therefore \arg\left(\frac{1+z}{1+\bar{z}}\right) = \arg\left(\frac{1+z}{1+\frac{1}{z}}\right) = \arg(z) = \theta.$$

27. (b) Let $z = x + iy, \bar{z} = x - iy$

$$\text{Now, } z = 1 - \bar{z}$$

$$\Rightarrow x + iy = 1 - (x - iy)$$

$$\Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$$

$$\text{Now, } |z| = 1 \Rightarrow x^2 + y^2 = 1 \Rightarrow y^2 = 1 - x^2$$

$$\Rightarrow y = \pm \frac{\sqrt{3}}{2}$$

$$\text{Now, } \tan \theta = \frac{y}{x} \quad (\theta \text{ is the argument})$$

$$= \frac{\sqrt{3}}{2} \div \frac{1}{2} \quad (+ve \text{ since only principal argument})$$

$$= \sqrt{3}$$

$$\Rightarrow \theta = \tan^{-1} \sqrt{3} = \frac{\pi}{3}$$

Hence, z is not a real number

So, statement-1 is false and 2 is true.

28. (a) Let $z = x + iy \Rightarrow z^2 = x^2 - y^2 + 2ixy$

$$\begin{aligned} \text{Now, } \frac{1+z^2}{2iz} &= \frac{1+x^2-y^2+2ixy}{2i(x+iy)} = \frac{(x^2-y^2+1)+2ixy}{2ix-2y} \\ &= \frac{(x^2-y^2+1)+2ixy}{-2y+2ix} \times \frac{-2y-2ix}{-2y-2ix} \\ &= \frac{y(x^2+y^2-1)+x(x^2+y^2+1)i}{2(x^2+y^2)} \end{aligned}$$

$$a = \frac{x(x^2+y^2+1)}{2(x^2+y^2)}$$

$$\text{Since, } |z| = 1 \Rightarrow \sqrt{x^2+y^2} = 1$$

$$\Rightarrow x^2+y^2 = 1$$

$$\therefore a = \frac{x(1+1)}{2 \times 1} = x$$

$$\text{Also } z \neq 1 \Rightarrow x + iy \neq 1$$

$$\therefore A = (-1, 1)$$

29. (d) Let $z_1 = 1 + i$ and $z_2 = 1 - i$

$$\frac{z_2}{z_1} = \frac{1-i}{1+i} = \frac{(1-i)(1-i)}{(1+i)(1-i)} = -i$$

$$\frac{2z_1 + 3z_2}{2z_1 - 3z_2} = \frac{2+3\left(\frac{z_2}{z_1}\right)}{2-3\left(\frac{z_2}{z_1}\right)} = \frac{2-3i}{2+3i}$$

$$\begin{aligned} \left| \frac{2z_1 + 3z_2}{2z_1 - 3z_2} \right| &= \left| \frac{2-3i}{2+3i} \right| = \left| \frac{2-3i}{2+3i} \right| \left[\because \left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|} \right] \\ &= \frac{\sqrt{4+9}}{\sqrt{4+9}} = 1 \end{aligned}$$

30. (b) $|z_1 + z_2|^2 + |z_1 - z_2|^2$

$$= |z_1|^2 + |z_2|^2 + 2|z_1||z_2| + |z_1|^2 + |z_2|^2 - 2|z_1||z_2|$$

$$= 2|z_1|^2 + 2|z_2|^2 = 2\left[|z_1|^2 + |z_2|^2\right]$$

31. (a) Let $|Z| = |W| = r$

$$\Rightarrow Z = re^{i\theta}, W = re^{i\phi}$$

where $\theta + \phi = \pi$

$$\therefore \bar{W} = re^{-i\phi}$$

$$\text{Now, } Z = re^{i(\pi-\phi)} = re^{i\pi} \times e^{-i\phi} = -re^{-i\phi}$$

$$= -\bar{W}$$

Thus, statement-1 is true but statement-2 is false.

32. (b) Statement - 1 and 2 both are true.

It is fundamental property.

But Statement - 2 is not correct explanation for Statement - 1.

33. (a) Let $z = x + iy$

$$|z-1| = |z+1| \Rightarrow (x-1)^2 + y^2 = (x+1)^2 + y^2$$

$$\Rightarrow x = 0 \Rightarrow \operatorname{Re} z = 0$$

$$|z-1| = |z-i| \Rightarrow (x-1)^2 + y^2 = x^2 + (y-1)^2$$

$$\Rightarrow x = y$$

$$|z+1| = |z-i| \Rightarrow (x+1)^2 + y^2 = x^2 + (y-1)^2$$

$$\Rightarrow x = -y$$

Only $(0, 0)$ will satisfy all conditions.

\Rightarrow Number of complex number $z = 1$

34. (c) $\left(\frac{1}{i-1}\right) = \frac{1}{(i-1)} = \frac{1}{-i-1} = \frac{-1}{i+1}$

35. (a) Given that $z^{\frac{1}{3}} = p + iq$

$$\Rightarrow z = p^3 + (iq)^3 + 3p(iq)(p+iq)$$

$$\Rightarrow x - iy = p^3 - 3pq^2 + i(3p^2q - q^3)$$

Comparing both side, we get

$$\therefore x = p^3 - 3pq^2 \Rightarrow \frac{x}{p} = p^2 - 3q^2 \quad \dots(i)$$

$$\text{and } y = q^3 - 3p^2q \Rightarrow \frac{y}{q} = q^2 - 3p^2 \quad \dots(ii)$$

Adding (i) and (ii), we get

$$\therefore \frac{x}{p} + \frac{y}{q} = -2p^2 - 2q^2 \therefore \left(\frac{x}{p} + \frac{y}{q}\right) / (p^2 + q^2) = -2$$

36. (c) Given that $\arg zw = \pi$

$$\Rightarrow \arg z + \arg w = \pi$$

...(i)

$$\bar{z} + i\bar{w} = 0 \Rightarrow \bar{z} = -i\bar{w}$$

Replace i by $-i$, we get

$$\therefore z = iw \Rightarrow \arg z = \frac{\pi}{2} + \arg w$$

$$\Rightarrow \arg z = \frac{\pi}{2} + \pi - \arg z \quad (\text{from (i)})$$

$$\therefore \arg z = \frac{3\pi}{4}$$

37. (b) Given that

$$\left(\frac{1+i}{1-i}\right)^x = 1 \Rightarrow \left[\frac{(1+i)^2}{1-i^2}\right]^x = 1$$

$$\left(\frac{1+i^2+2i}{1+1}\right)^x = 1 \Rightarrow (i)^x = 1; \therefore x = 4n; \quad n \in I^+$$

38. (a) $|\bar{z}\omega| = |\bar{z}| |\omega| = |z| |\omega| = z\omega = 1$ [$\because |\bar{z}| = |z|$]

$$\operatorname{Arg}(\bar{z}\omega) = \arg(\bar{z}) + \arg(\omega)$$

$$= -\arg(z) + \arg \omega = -\frac{\pi}{2}$$

$$[\because \arg(\bar{z}) = -\arg(z)]$$

$$\therefore \bar{z}\omega = -1$$

39. (c) Given that $|z-4| < |z-2|$

$$\text{Let } z = x + iy$$

$$\Rightarrow |(x-4) + iy| < |(x-2) + iy|$$

$$\Rightarrow (x-4)^2 + y^2 < (x-2)^2 + y^2$$

$$\Rightarrow x^2 - 8x + 16 < x^2 - 4x + 4 \Rightarrow 12 < 4x$$

$$\Rightarrow x > 3 \Rightarrow \operatorname{Re}(z) > 3$$

40. (b) Let $|z| = |\omega| = r$

$$\therefore z = re^{i\theta}, \omega = re^{i\phi} \quad \text{where } \theta + \phi = \pi.$$

$$\therefore z = re^{i(\pi-\phi)} = re^{i\pi} \cdot e^{-i\phi} = -re^{-i\phi} = -\bar{\omega}.$$

$$[\because e^{i\pi} = -1 \text{ and } \bar{\omega} = re^{-i\phi}]$$

41. (c) Let $z = x + iy$

$$\therefore z^2 = i|z|^2$$

$$\therefore x^2 - y^2 + 2ixy = i(x^2 + y^2)$$

$$\Rightarrow x^2 - y^2 = 0 \text{ and } 2xy = x^2 + y^2$$

$$\Rightarrow (x-y)(x+y) = 0 \text{ and } (x-y)^2 = 0$$

$$\Rightarrow x = y$$

42. (a) Given that, $\alpha = \frac{-1 + \sqrt{3}i}{2} = \omega$

$$\therefore (2 + \omega)^4 = a + b\omega \Rightarrow (4 + \omega^2 + 4\omega)^2 = a + b\omega$$

$$\Rightarrow (\omega^2 + 4(1 + \omega))^2 = a + b\omega$$

$$\Rightarrow (\omega^2 - 4\omega^2)^2 = a + b\omega$$

$$[\because 1 + \omega = -\omega^2]$$

$$\Rightarrow (-3\omega^2)^2 = a + b\omega \Rightarrow 9\omega^4 = a + b\omega$$

$$\Rightarrow 9\omega = a + b\omega \quad (\because \omega^3 = 1)$$

On comparing, $a = 0, b = 9$

$$\Rightarrow a + b = 0 + 9 = 9.$$

43. (c) $\left(\frac{1 + \cos \frac{5\pi}{18} + i \sin \frac{5\pi}{18}}{1 + \cos \frac{5\pi}{18} - i \sin \frac{5\pi}{18}}\right)^3$

$$= \left(\frac{2 \cos^2 \frac{5\pi}{36} + i 2 \sin \frac{5\pi}{36} \cdot \cos \frac{5\pi}{36}}{2 \cos^2 \frac{5\pi}{36} - i 2 \sin \frac{5\pi}{36} \cdot \cos \frac{5\pi}{36}}\right)^3$$

$$= \left(\frac{\cos \frac{5\pi}{36} + i \sin \frac{5\pi}{36}}{\cos \frac{5\pi}{36} - i \sin \frac{5\pi}{36}} \right)^3 = \left(\cos \frac{5\pi}{36} + i \sin \frac{5\pi}{36} \right)^6$$

$$= \cos \left(6 \times \frac{5\pi}{36} \right) + i \sin \left(6 \times \frac{5\pi}{36} \right) = \cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}$$

$$= -\frac{\sqrt{3}}{2} + i \frac{1}{2} = -\frac{1}{2}(\sqrt{3} - i)$$

44. (b) $3 + 2\sqrt{-54} = 3 + 6\sqrt{6}i$

Let $\sqrt{3 + 6\sqrt{6}i} = a + ib$

$$\Rightarrow a^2 - b^2 = 3 \text{ and } ab = 3\sqrt{6}$$

$$\Rightarrow a^2 + b^2 = \sqrt{(a^2 - b^2)^2 + 4a^2b^2} = 15$$

So, $a = \pm 3$ and $b = \pm\sqrt{6}$

$$\sqrt{3 + 6\sqrt{6}i} = \pm(3 + \sqrt{6}i)$$

Similarly, $\sqrt{3 - 6\sqrt{6}i} = \pm(3 - \sqrt{6}i)$

$$\text{Im}(\sqrt{3 + 6\sqrt{6}i} - \sqrt{3 - 6\sqrt{6}i}) = \pm 2\sqrt{6}$$

45. (b) Let $\alpha = \omega, b = 1 + \omega^3 + \omega^6 + \dots = 101$
 $a = (1 + \omega)(1 + \omega^2 + \omega^4 + \dots + \omega^{198} + \omega^{200})$

$$= (1 + \omega) \frac{(1 - (\omega^2)^{101})}{1 - \omega^2} = \frac{(\omega + 1)(\omega^{202} - 1)}{(\omega^2 - 1)}$$

$$\Rightarrow a = \frac{(1 + \omega)(1 - \omega)}{1 - \omega^2} = 1$$

Required equation = $x^2 - (101 + 1)x + (101) \times 1 = 0$

$$\Rightarrow x^2 - 102x + 101 = 0$$

46. (d) $\because z = x + iy$

$$\left(\frac{z-1}{2z+i} \right) = \frac{(x-1)+iy}{2(x+iy)+i}$$

$$= \frac{(x-1)+iy}{2x+(2y+1)i} \times \frac{2x-(2y+1)i}{2x-(2y+1)i}$$

$$\text{Re} \left(\frac{z+1}{2z+i} \right) = \frac{2x(x-1) + y(2y+1)}{(2x)^2 + (2y+1)^2} = 1$$

$$\Rightarrow \left(x + \frac{1}{2} \right)^2 + \left(y + \frac{3}{4} \right)^2 = \left(\frac{\sqrt{5}}{4} \right)^2$$

47. (d) $\frac{\sqrt{3}}{2} + \frac{i}{2} = -i \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i \right) = -i\omega$

where ω is imaginary cube root of unity.

Now, $(1 + iz + z^5 + iz^8)^9$

$$= (1 + \omega - i\omega^2 + i\omega^2)^9 = (1 + \omega)^9$$

$$= (-\omega^2)^9 = -\omega^{18} = -1$$

$$(\because 1 + \omega + \omega^2 = 0)$$

48. (a) $-(6 + i)^3 = x + iy$

$$\Rightarrow -[216 + i^3 + 18i(6 + i)] = x + iy$$

$$\Rightarrow -[216 - i + 108i - 18] = x + iy$$

$$\Rightarrow -216 + i - 108i + 18 = x + iy$$

$$\Rightarrow -198 - 107i = x + iy$$

$$\Rightarrow x = -198, y = -107$$

$$\Rightarrow y - x = -107 + 198 = 91$$

49. (a) $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2} \right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2} \right)^5$

$$= \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)^5 + \left(\cos \frac{\pi}{6} - i \sin \frac{\pi}{6} \right)^5$$

$$= \left(e^{i\frac{\pi}{6}} \right)^5 + \left(e^{-i\frac{\pi}{6}} \right)^5 = 2 \cos \frac{\pi}{6} = \sqrt{3}$$

$$\Rightarrow \text{Im}(z) = 0, \text{Re}(z) = \sqrt{3}$$

50. (d) Let $l = \left(\frac{1 + i\sqrt{3}}{1 - i\sqrt{3}} \right)$.

$$\therefore l = \left(\frac{1 + i\sqrt{3}}{1 - i\sqrt{3}} \right) \times \left(\frac{1 + i\sqrt{3}}{1 + i\sqrt{3}} \right)$$

$$= \left(\frac{-2 + i2\sqrt{3}}{4} \right) = \left(\frac{1 - i\sqrt{3}}{-2} \right)$$

Also, $l = \left(\frac{1 + i\sqrt{3}}{1 - i\sqrt{3}} \right) \times \left(\frac{1 - i\sqrt{3}}{1 - i\sqrt{3}} \right)$

$$= \left(\frac{4}{-2 - i2\sqrt{3}} \right) = \left(\frac{-2}{1 + i\sqrt{3}} \right)$$

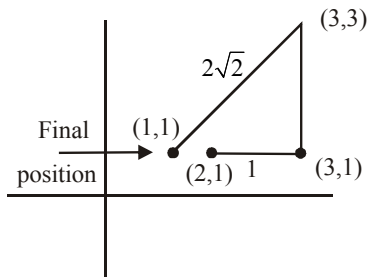
Now, $\left(\frac{1 + i\sqrt{3}}{1 - i\sqrt{3}} \right)^3 = \left(\frac{1 + i\sqrt{3}}{1 - i\sqrt{3}} \right) \times \left(\frac{1 + i\sqrt{3}}{1 - i\sqrt{3}} \right) \times \left(\frac{1 + i\sqrt{3}}{1 - i\sqrt{3}} \right)$

$$= \left(\frac{1 + i\sqrt{3}}{1 - i\sqrt{3}} \right) \times \left(\frac{-2}{1 + i\sqrt{3}} \right) \times \left(\frac{1 - i\sqrt{3}}{-2} \right) = 1$$

\therefore least positive integer n is 3.



51. (a)



So new position is at the point $1 + i$

52. (a)

$$\left| \frac{z_1 - 2z_2}{2 - z_1\bar{z}_2} \right| = 1$$

$$\Rightarrow |z_1 - 2z_2|^2 = |2 - z_1\bar{z}_2|^2$$

$$\Rightarrow (z_1 - 2z_2)(\overline{z_1 - 2z_2}) = (2 - z_1\bar{z}_2)(\overline{2 - z_1\bar{z}_2})$$

$$\Rightarrow (z_1 - 2z_2)(\bar{z}_1 - 2\bar{z}_2) = (2 - z_1\bar{z}_2)(2 - \bar{z}_1z_2)$$

$$\Rightarrow (z_1\bar{z}_1) - 2z_1\bar{z}_2 - 2\bar{z}_1z_2 + 4z_2\bar{z}_2$$

$$= 4 - 2\bar{z}_1z_2 - 2z_1\bar{z}_2 + z_1\bar{z}_1z_2\bar{z}_2$$

$$\Rightarrow |z_1|^2 + 4|z_2|^2 = 4 + |z_1|^2|z_2|^2$$

$$\Rightarrow |z_1|^2 + 4|z_2|^2 - 4 - |z_1|^2|z_2|^2 = 0$$

$$(|z_1|^2 - 4)(1 - |z_2|^2) = 0$$

$\therefore |z_2| \neq 1$

$\therefore |z_1|^2 = 4$

$\Rightarrow |z_1| = 2$

\Rightarrow Point z_1 lies on circle of radius 2.

53. (a)

$$\frac{z^2}{z-1} = \frac{\bar{z}^2}{\bar{z}-1} \quad \left[\because \left(\frac{z_1}{z_2} \right) = \frac{\bar{z}_1}{\bar{z}_2} \right]$$

$$\Rightarrow z\bar{z}z - z^2 = z.\bar{z}.\bar{z} - \bar{z}^2$$

$$\Rightarrow |z|^2.z - z^2 = |z|^2.\bar{z} - \bar{z}^2$$

$$\Rightarrow |z|^2(z - \bar{z}) - (z - \bar{z})(z + \bar{z}) = 0$$

$$\Rightarrow (z - \bar{z})(|z|^2 - (z + \bar{z})) = 0$$

Either $z - \bar{z} = 0$ or $|z|^2 - (z + \bar{z}) = 0$

Either $z = \bar{z} \Rightarrow$ real axis

or $|z|^2 = z + \bar{z} \Rightarrow z\bar{z} - z - \bar{z} = 0$

represents a circle passing through origin.

54. (a) $(1 + \omega)^7 = A + B\omega$
 $(-\omega^2)^7 = A + B\omega$ ($\because \omega^{14} = \omega^{12}.\omega^2 = \omega^2$)
 $-\omega^2 = A + B\omega$
 $1 + \omega = A + B\omega$
 $\Rightarrow A = 1, B = 1.$
55. (a) $|z + 1| = |z + 4 - 3| \leq |z + 4| + |-3| \leq |3| + |-3|$
 $\Rightarrow |z + 1| \leq 6 \Rightarrow |z + 1|_{\max} = 6$

56. (c) Given that $w = \frac{z}{z - \frac{1}{3}i}$

$$\Rightarrow |w| = \frac{|z|}{|z - \frac{1}{3}i|} = 1 \quad \left[\because \left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|} \right]$$

$$\Rightarrow |z| = \left| z - \frac{1}{3}i \right|$$

\Rightarrow distance of z from origin and point $\left(0, \frac{1}{3} \right)$ is same

hence z lies on bisector of the line joining points $(0, 0)$ and $(0, 1/3)$.
Hence z lies on a straight line.

57. (c) $|z_1 + z_2| = |z_1| + |z_2| \Rightarrow z_1$ and z_2 are collinear and are to the same side of origin; hence $\arg z_1 - \arg z_2 = 0$.
58. (c) $\because (x-1)^3 + 8 = 0 \Rightarrow (x-1) = (-2) (1)^{1/3}$
 $\Rightarrow x - 1 = -2$ or -2ω or $-2\omega^2$
or $x = -1$ or $1 - 2\omega$ or $1 - 2\omega^2$.
59. (b) Given that $|z^2 - 1| = |z|^2 + 1 \Rightarrow |z^2 - 1|^2 = (z\bar{z} + 1)^2$
 $[\because |z|^2 = z\bar{z}]$

$$\Rightarrow (z^2 - 1)(\bar{z}^2 - 1) = (z\bar{z} + 1)^2 \quad (\because \overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2)$$

$$\Rightarrow z^2\bar{z}^2 - z^2 - \bar{z}^2 + 1 = z^2\bar{z}^2 + 2z\bar{z} + 1$$

$$\Rightarrow z^2 + 2z\bar{z} + \bar{z}^2 = 0$$

$$\Rightarrow (z + \bar{z})^2 = 0 \Rightarrow z = -\bar{z}$$

$\Rightarrow z$ is purely imaginary

60. (b) Let the circle be $|z - z_0| = r$. Then according to given conditions $|z_0 - z_1| = r + a$... (i)
 $|z_0 - z_2| = r + b$... (ii)
- Subtract (ii) from (i)
- we get $|z_0 - z_1| - |z_0 - z_2| = a - b$.
- \therefore Locus of centre z_0 is $|z - z_1| - |z - z_2| = a - b$, which represents a hyperbola.

61. (a) $\because \alpha + \beta = 64, \alpha\beta = 256$

$$\frac{\alpha^{3/8} + \beta^{3/8}}{\beta^{5/8} + \alpha^{5/8}} = \frac{\alpha + \beta}{(\alpha\beta)^{5/8}} = \frac{64}{(2^8)^{5/8}} = \frac{64}{32} = 2$$

62. (b) Let α and β be the roots of the given quadratic equation,

$$2x^2 + 2x - 1 = 0 \quad \dots(i)$$

Then, $\alpha + \beta = -\frac{1}{2} \Rightarrow -1 = 2\alpha + 2\beta$

and $4\alpha^2 + 2\alpha - 1 = 0$ [$\because \alpha$ is root of eq. (i)]

$$\Rightarrow 4\alpha^2 + 2\alpha + 2\alpha + 2\beta = 0 \Rightarrow \beta = -2\alpha(\alpha + 1)$$

63. (b) Let $|x| = y$ then

$$9y^2 - 18y + 5 = 0$$

$$\Rightarrow 9y^2 - 15y - 3y + 5 = 0$$

$$\Rightarrow (3y - 1)(3y - 5) = 0$$

$$\Rightarrow y = \frac{1}{3} \text{ or } \frac{5}{3} \Rightarrow |x| = \frac{1}{3} \text{ or } \frac{5}{3}$$

Roots are $\pm \frac{1}{3}$ and $\pm \frac{5}{3}$

$$\therefore \text{Product} = \frac{25}{81}$$

64. (d) Let α and β be the roots of the quadratic equation

$$7x^2 - 3x - 2 = 0$$

$$\therefore \alpha + \beta = \frac{3}{7}, \alpha\beta = \frac{-2}{7}$$

Now, $\frac{\alpha}{1 - \alpha^2} + \frac{\beta}{1 - \beta^2}$

$$= \frac{\alpha - \alpha\beta(\alpha + \beta) + \beta}{1 - (\alpha^2 + \beta^2) + (\alpha\beta)^2}$$

$$= \frac{(\alpha + \beta) - \alpha\beta(\alpha + \beta)}{1 - (\alpha + \beta)^2 + 2\alpha\beta + (\alpha\beta)^2}$$

$$= \frac{\frac{3}{7} + \frac{2}{7} \times \frac{3}{7}}{1 - \frac{9}{49} + 2 \times \frac{-2}{7} + \frac{4}{49}} = \frac{27}{16}$$

65. (d) $u = \frac{2(x + iy) + i}{(x + iy) - ki} = \frac{2x + i(2y + 1)}{x + i(y - k)}$

Real part of $u = \text{Re}(u) = \frac{2x^2 + (y - K)(2y + 1)}{x^2 + (y - K)^2}$

Imaginary part of u

$$= \text{Im}(u) = \frac{-2x(y - K) + x(2y + 1)}{x^2 + (y - K)^2}$$

$$\therefore \text{Re}(u) + \text{Im}(u) = 1$$

$$\Rightarrow 2x^2 + 2y^2 - 2Ky + y - K - 2xy + 2Kx + 2xy + x$$

$$= x^2 + y^2 + K^2 - 2Ky$$

Since, the curve intersect at y -axis

$$\therefore x = 0$$

$$\Rightarrow y^2 + y - K(K + 1) = 0$$

Let y_1 and y_2 are roots of equations if $x = 0$

$$\therefore y_1 + y_2 = -1$$

$$y_1 y_2 = -(K^2 + K)$$

$$\therefore (y_1 - y_2)^2 = (1 + 4K^2 + 4K)$$

Given $PQ = 5 \Rightarrow |y_1 - y_2| = 5$

$$\Rightarrow 4K^2 + 4K - 24 = 0 \Rightarrow K = 2 \text{ or } -3$$

as $K > 0, \therefore K = 2$

66. (b) Since α is common root of $x^2 - x + 2\lambda = 0$ and

$$3x^2 - 10x + 27\lambda = 0$$

$$\therefore 3\alpha^2 - 10\alpha + 27\lambda = 0 \quad \dots(i)$$

$$3\alpha^2 - 3\alpha + 6\lambda = 0 \quad \dots(ii)$$

\therefore On subtract, we get $\alpha = 3\lambda$

Now, $\alpha\beta = 2\lambda \Rightarrow 3\lambda \cdot \beta = 2\lambda \Rightarrow \beta = \frac{2}{3}$

$$\Rightarrow \alpha + \beta = 1 \Rightarrow 3\lambda + \frac{2}{3} = 1 \Rightarrow \lambda = \frac{1}{9} \text{ and}$$

$$\alpha\gamma = 9\lambda \Rightarrow 3\lambda \cdot \gamma = 9\lambda \Rightarrow \gamma = 3$$

$$\therefore \frac{\beta\gamma}{\lambda} = 18$$

67. (d) $\alpha \cdot \beta = 2$ and $\alpha + \beta = -p$ also $\frac{1}{\alpha} + \frac{1}{\beta} = -q$

$$\Rightarrow p = 2q$$

Now $\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$

$$= \left[\alpha\beta + \frac{1}{\alpha\beta} - \frac{\alpha}{\beta} - \frac{\beta}{\alpha}\right]\left[\alpha\beta + \frac{1}{\alpha\beta} + 2\right]$$

$$= \frac{9}{2} \left[\frac{5}{2} - \frac{\alpha^2 + \beta^2}{2} \right] = \frac{9}{4} [5 - (p^2 - 4)]$$

$$= \frac{9}{4} (9 - p^2) \quad [\because \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta]$$

68. (c) The given quadratic equation is

$$(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$$

\therefore One root is in the interval (0, 1)

$$\therefore f(0)f(1) \leq 0$$

$$\Rightarrow 2(\lambda^2 + 1 - 4\lambda + 2) \leq 0$$

$$\Rightarrow 2(\lambda^2 - 4\lambda + 3) \leq 0$$

$$(\lambda - 1)(\lambda - 3) \leq 0 \Rightarrow \lambda \in [1, 3]$$

But at $\lambda = 1$, both roots are 1 so $\lambda \neq 1$

$$\therefore \lambda \in (1, 3]$$

69. (c) Since, α and β are the roots of the equation $5x^2 + 6x - 2 = 0$

$$\text{Then, } 5\alpha^2 + 6\alpha - 2 = 0, \quad 5\beta^2 + 6\beta - 2 = 0$$

$$5\alpha^2 + 6\alpha = 2$$

$$5S_6 + 6S_5 = 5(\alpha^6 + \beta^6) + 6(\alpha^5 + \beta^5)$$

$$= (5\alpha^4 + 6\alpha^5) + (5\beta^6 + 6\beta^5)$$

$$= \alpha^4(5\alpha^2 + 6\alpha) + \beta^4(5\beta^2 + 6\beta)$$

$$= 2(\alpha^4 + \beta^4) = 2S_4$$

70. (a) Let $e^x = t \in (0, \infty)$

Given equation

$$t^4 + t^3 - 4t^2 + t + 1 = 0$$

$$\Rightarrow t^2 + t - 4 + \frac{1}{t} + \frac{1}{t^2} = 0$$

$$\Rightarrow \left(t^2 + \frac{1}{t^2} \right) + \left(t + \frac{1}{t} \right) - 4 = 0$$

$$\text{Let } t + \frac{1}{t} = y$$

$$(y^2 - 2) + y - 4 = 0 \Rightarrow y^2 + y - 6 = 0$$

$$y^2 + y - 6 = 0 \Rightarrow y = -3, 2$$

$$\Rightarrow y = 2 \Rightarrow t + \frac{1}{t} = 2$$

$$\Rightarrow e^x + e^{-x} = 2$$

$x = 0$, is the only solution of the equation

Hence, there only one solution of the given equation.

71. (8) Since, $2x^2 + (a - 10)x + \frac{33}{2} = 2a$ has real roots,

$$\therefore D \geq 0$$

$$\Rightarrow (a - 10)^2 - 4(2) \left(\frac{33}{2} - 2a \right) \geq 0$$

$$\Rightarrow (a - 10)^2 - 4(33 - 4a) \geq 0$$

$$\Rightarrow a^2 - 4a - 32 \geq 0$$

$$\Rightarrow (a - 8)(a + 4) \geq 0$$

$$\Rightarrow a \leq -4 \cup a \geq 8$$

$$\Rightarrow a \in (-\infty, -4] \cup [8, \infty)$$

72. (a) Let $z = \alpha \pm i\beta$ be the complex roots of the equation

So, sum of roots = $2\alpha = -b$ and

Product of roots = $\alpha^2 + \beta^2 = 45$

$$(\alpha + 1)^2 + \beta^2 = 40$$

$$\text{Given, } |z + 1| = 2\sqrt{10}$$

$$\Rightarrow (\alpha + 1)^2 - \alpha^2 = -5 \quad [\because \beta^2 = 45 - \alpha^2]$$

$$\Rightarrow 2\alpha + 1 = -5 \Rightarrow 2\alpha = -6$$

Hence, $b = 6$ and $b^2 - b = 30$

73. (d) $\alpha^5 = 5\alpha + 3$

$$\beta^5 = 5\beta + 3$$

$$p_5 = 5(\alpha + \beta) + 6 = 5(1) + 6$$

$$[\because \text{from } x^2 - x - 1 = 0, \alpha + \beta = \frac{-b}{a} = 1]$$

$$p_5 = 11 \text{ and } p_5 = \alpha^2 + \beta^2 = \alpha + 1 + \beta + 1$$

$$p_2 = 3 \text{ and } p_3 = \alpha^3 + \beta^3 = 2\alpha + 1 + 2\beta + 1$$

$$= 2(1) + 2 = 4$$

$$p_2 \times p_3 = 12 \text{ and } p_5 = 11 \Rightarrow p_5 \neq p_2 \times p_3$$

74. (b) $(k + 1)\tan^2 x - \sqrt{2}\lambda \tan x + (k - 1) = 0$

$$\tan \alpha + \tan \beta = \frac{\sqrt{2}\lambda}{k + 1} \quad [\text{Sum of roots}]$$

$$\tan \alpha \cdot \tan \beta = \frac{k - 1}{k + 1} \quad [\text{Product of roots}]$$

$$\therefore \tan(\alpha + \beta) = \frac{\frac{\sqrt{2}\lambda}{k + 1}}{1 - \frac{k - 1}{k + 1}} = \frac{\sqrt{2}\lambda}{2} = \frac{\lambda}{\sqrt{2}}$$

$$\tan^2(\alpha + \beta) = \frac{\lambda^2}{2} = 50$$

$$\lambda = 10.$$

75. (b) Given equation is, $x^2 + x \sin \theta - 2 \sin \theta = 0$

$$\alpha + \beta = -\sin \theta \text{ and } \alpha\beta = -2 \sin \theta$$

$$\frac{(\alpha^{12} + \beta^{12})\alpha^{12}\beta^{12}}{(\alpha^{12} + \beta^{12})(\alpha - \beta)^{24}} = \frac{(\alpha\beta)^{12}}{(\alpha - \beta)^{24}}$$

$$\therefore |\alpha - \beta| = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta} = \sqrt{\sin^2 \theta + 8\sin \theta}$$

$$\therefore \frac{(\alpha\beta)^{12}}{(\alpha - \beta)^{24}} = \frac{(2\sin \theta)^{12}}{\sin^{12} \theta (\sin \theta + 8)^{12}} = \frac{2^{12}}{(\sin \theta + 8)^{12}}$$

76. (d) Let $2^x - 1 = t$

$$5 + |t| = (t + 1)(t - 1) \Rightarrow |t| = t^2 - 6$$

When $t > 0$, $t^2 - t - 6 = 0 \Rightarrow t = 3$ or -2

$t = -2$ (rejected)

When $t < 0$, $t^2 + t - 6 = 0 \Rightarrow t = -3$ or 2 (both rejected)

$$\therefore 2^x - 1 = 3 \Rightarrow 2^x = 4 \Rightarrow x = 2$$

77. (d) Since $2 - \sqrt{3}$ is a root of the quadratic equation $x^2 + px + q = 0$

$\therefore 2 + \sqrt{3}$ is the other root

$$\Rightarrow x^2 + px + q = [x - (2 - \sqrt{3})][x - (2 + \sqrt{3})]$$

$$= x^2 - (2 + \sqrt{3})x - (2 - \sqrt{3})x + (2^2 - (\sqrt{3})^2)$$

$$= x^2 - 4x + 1$$

Now, by comparing $p = -4$, $q = 1$

$$\Rightarrow p^2 - 4q - 12 = 16 - 4 - 12 = 0$$

78. (c) Sum of roots $= \frac{3}{m^2 + 1}$

\therefore sum of roots is greatest. $\therefore m = 0$

Hence equation becomes $x^2 - 3x + 1 = 0$

Now, $\alpha + \beta = 3$, $\alpha\beta = 1 \Rightarrow |-\alpha - \beta| = \sqrt{5}$

$$|\alpha^3 - \beta^3| = |(\alpha - \beta)(\alpha^2 + \beta^2 + \alpha\beta)| = \sqrt{5}(9 - 1) = 8\sqrt{5}$$

79. (d) Let $\sqrt{x} = a$

\therefore given equation will become:

$$|a - 2| + a(a - 4) + 2 = 0$$

$$\Rightarrow |a - 2| + a^2 - 4a + 4 - 2 = 0$$

$$\Rightarrow |a - 2| + (a - 2)^2 - 2 = 0$$

Let $|a - 2| = y$ (Clearly $y \geq 0$)

$$\Rightarrow y + y^2 - 2 = 0$$

$$\Rightarrow y = 1 \text{ or } -2 \text{ (rejected)}$$

$$\Rightarrow |a - 2| = 1 \Rightarrow a = 1, 3$$

When $\sqrt{x} = 1 \Rightarrow x = 1$

When $\sqrt{x} = 3 \Rightarrow x = 9$

Hence, the required sum of solutions of the equation = 10

80. (c) The given quadratic equation is $x^2 - 2x + 2 = 0$

Then, the roots of the this equation are $\frac{2 \pm \sqrt{-4}}{2} = 1 \pm i$

Now, $\frac{\alpha}{\beta} = \frac{1-i}{1+i} = \frac{(1-i)^2}{1-i^2} = i$

or $\frac{\alpha}{\beta} = \frac{1-i}{1+i} = \frac{(1-i)^2}{1-i^2} = -i$ So, $\frac{\alpha}{\beta} = \pm i$

Now, $\left(\frac{\alpha}{\beta}\right)^n = 1 \Rightarrow (\pm i)^n = 1$

$\Rightarrow n$ must be a multiple of 4.

Hence, the required least value of $n = 4$.

81. (b) Let roots of the quadratic equation are α, β .

Given, $\lambda = \frac{\alpha}{\beta}$ and $\lambda + \frac{1}{\lambda} = 1 \Rightarrow \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = 1$

$$\frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} = 1 \dots (i)$$

The quadratic equation is, $3m^2x^2 + m(m-4)x + 2 = 0$

$$\therefore \alpha + \beta = \frac{m(4-m)}{3m^2} = \frac{4-m}{3m} \text{ and } \alpha\beta = \frac{2}{3m^2}$$

Put these values in eq (1),

$$\frac{\left(\frac{4-m}{3m}\right)^2 - \frac{2}{3m^2}}{\frac{2}{3m^2}} = 3$$

$$\Rightarrow (m-4)^2 = 18 \Rightarrow m = 4 \pm \sqrt{18}$$

Therefore, least value is $4 - \sqrt{18} = 4 - 3\sqrt{2}$

82. (d) Let α and β be the roots of the equation,

$$81x^2 + kx + 256 = 0$$

Given $(\alpha)^3 = \beta \Rightarrow \alpha = \beta^3$

\therefore Product of the roots $= \frac{256}{81}$

$$\therefore (\alpha)(\beta) = \frac{256}{81}$$

$$\Rightarrow \beta^4 = \left(\frac{4}{3}\right)^4 \Rightarrow \beta = \frac{4}{3} \Rightarrow \alpha = \frac{64}{27}$$

\therefore Sum of the roots $= -\frac{k}{81}$

$$\therefore \alpha + \beta = -\frac{k}{81} \Rightarrow \frac{4}{3} + \frac{64}{27} = -\frac{k}{81}$$

$$\Rightarrow k = -300$$

83. (d) Consider the quadratic equation

$$(c-5)x^2 - 2cx + (c-4) = 0$$

Now, $f(0), f(3) > 0$ and $f(0), f(2) < 0$

$$\Rightarrow (c-4)(4c-49) > 0 \text{ and } (c-4)(c-24) < 0$$

$$\Rightarrow c \in (-\infty, 4) \cup \left(\frac{49}{4}, \infty\right) \text{ and } c \in (4, 24)$$

$$\Rightarrow c \in \left(\frac{49}{4}, 24\right)$$

Integral values in the interval $\left(\frac{49}{4}, 24\right)$ are 13, 14, ..., 23.

$$\therefore S = \{13, 14, \dots, 23\}$$

84. (d) The given quadratic equation is

$$x^2 + (3-\lambda)x + 2 = \lambda$$

$$\text{Sum of roots} = \alpha + \beta = \lambda - 3$$

$$\text{Product of roots} = \alpha\beta = 2 - \lambda$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= (\lambda - 3)^2 - 2(2 - \lambda)$$

$$= \lambda^2 - 4\lambda + 5$$

$$= (\lambda - 2)^2 + 1$$

$$\text{For least } (\alpha^2 + \beta^2) \lambda = 2.$$

85. (a) Consider the equation

$$x^2 + 2x + 2 = 0$$

$$x = \frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm i$$

$$\text{Let } \alpha = -1 + i, \beta = -1 - i$$

$$\alpha^{15} + \beta^{15} = (-1 + i)^{15} + (-1 - i)^{15}$$

$$= \left(\sqrt{2}e^{i\frac{3\pi}{4}}\right)^{15} + \left(\sqrt{2}e^{-i\frac{3\pi}{4}}\right)^{15}$$

$$= (\sqrt{2})^{15} \left[e^{i\frac{45\pi}{4}} + e^{-i\frac{45\pi}{4}} \right]$$

$$= (\sqrt{2})^{15} \cdot 2 \cos \frac{45\pi}{4} = (\sqrt{2})^{15} \cdot 2 \cos \frac{3\pi}{4}$$

$$= \frac{-2}{\sqrt{2}} (\sqrt{2})^{15}$$

$$= -2(\sqrt{2})^{14} = -256$$

86. (a) The roots of $6x^2 - 11x + \alpha = 0$ are rational numbers.

\therefore Discriminant D must be perfect square number.

$$D = (-11)^2 - 4 \cdot 6 \cdot \alpha$$

$$= 121 - 24\alpha \text{ must be a perfect square}$$

Hence, possible values for α are

$$\alpha = 3, 4, 5.$$

\therefore 3 positive integral values are possible.

87. (b) Given quadratic equation is: $x^2 - mx + 4 = 0$

Both the roots are real and distinct.

So, discriminant $B^2 - 4AC > 0$.

$$\therefore m^2 - 4 \cdot 1 \cdot 4 > 0$$

$$\therefore (m-4)(m+4) > 0$$

$$\therefore m \in (-\infty, -4) \cup (4, \infty) \dots(i)$$

Since, both roots lies in $[1, 5]$

$$\therefore -\frac{-m}{2} \in (1, 5)$$

$$\Rightarrow m \in (2, 10) \dots(ii)$$

$$\text{And } 1 \cdot (1 - m + 4) > 0 \Rightarrow m < 5$$

$$\therefore m \in (-\infty, 5) \dots(iii)$$

$$\text{And } 1 \cdot (25 - 5m + 4) > 0 \Rightarrow m < \frac{29}{5}$$

$$\therefore m \in \left(-\infty, \frac{29}{5}\right) \dots(iv)$$

From (i), (ii), (iii) and (iv), $m \in (4, 5)$

88. (a) $\therefore z_0$ is a root of quadratic equation

$$x^2 + x + 1 = 0$$

$$\therefore z_0 = \omega \text{ or } \omega^2 \Rightarrow z_0^3 = 1$$

$$\therefore z = 3 + 6iz_0^{81} - 3iz_0^{93}$$

$$= 3 + 6i((z_0^3)^{27}) - 3i((z_0^3)^{31})$$

$$= 3 + 6i - 3i$$

$$= 3 + 3i$$

$$\therefore \arg(z) \tan^{-1}\left(\frac{3}{3}\right) = \frac{\pi}{4}$$

89. (b) $\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$

$$\frac{x+p+x+q}{(x+p)(x+q)} = \frac{1}{r}$$

$$(2x+p+q)r = x^2 + px + qx + pq$$

$$x^2 + (p+q-2r)x + pq - pr - qr = 0$$

Let α and β be the roots.

$$\therefore \alpha + \beta = -(p + q - 2r) \quad \dots (i)$$

$$\& \alpha\beta = pq - pr - qr \quad \dots (ii)$$

$$\therefore \alpha = -\beta \text{ (given)}$$

\(\therefore\) in eq. (1), we get

$$\Rightarrow -(p + q - 2r) = 0 \quad \dots (iii)$$

$$\text{Now, } \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= -(p + q - 2r)^2 - 2(pq - pr - qr) \dots \text{(from (i) and (ii))}$$

$$= p^2 + q^2 + 4r^2 + 2pq - 4pr - 4qr - 2pq + 2pr + 2qr$$

$$= p^2 + q^2 + 4r^2 - 2pr - 2qr$$

$$= p^2 + q^2 + 2r(2r - p - q) \quad \dots \text{(from (iii))}$$

$$= p^2 + q^2 + 0$$

$$= p^2 + q^2$$

90. (b) Here, $9x^2 + 27x + 20 = 0$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-27 \pm \sqrt{27^2 - 4 \times 9 \times 20}}{2 \times 9}$$

$$\Rightarrow x = -\frac{4}{3}, -\frac{5}{3}$$

Given, $\cos A = -\frac{3}{5}$

$$\therefore \sec A = \frac{1}{\cos A} = -\frac{5}{3}$$

Here, A is an obtuse angle.

$$\therefore \tan A = -\sqrt{\sec^2 A - 1} = -\frac{4}{3}$$

Hence, roots of the equation are $\sec A$ and $\tan A$.

91. (b) As $\tan A$ and $\tan B$ are the roots of $3x^2 - 10x - 25 = 0$,

$$\text{So, } \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{10}{3}}{1 + \frac{25}{3}} = \frac{10/3}{28/3} = \frac{5}{14}$$

Now, $\cos 2(A+B) = -1 + 2 \cos^2(A+B)$

$$= \frac{1 - \tan^2(A+B)}{1 + \tan^2(A+B)} \Rightarrow \cos^2(A+B) = \frac{196}{221}$$

$$\therefore 3\sin^2(A+B) - 10\sin(A+B)\cos(A+B) - 25\cos^2(A+B) = \cos^2(A+B)[3\tan^2(A+B) - 10\tan(A+B) - 25]$$

$$= \frac{75 - 700 - 4900}{196} \times \frac{196}{221} = -\frac{5525}{196} \times \frac{196}{221} = -25$$

92. (d) If a and -1 are the roots of the polynomial, then we get

$$f(x) = x^2 + (1-a)x - a$$

$$\therefore f(1) = 2 - 2a$$

and $f(2) = 6 - 3a$

$$\text{As, } f(1) + f(2) = 0 \Rightarrow 2 - 2a + 6 - 3a = 0 \Rightarrow a = \frac{8}{5}$$

Therefore, the other root is $\frac{8}{5}$

93. (b) α, β are roots of $x^2 - x + 1 = 0$

$$\therefore \alpha = -\omega \text{ and } \beta = -\omega^2$$

where ω is cube root of unity

$$\therefore \alpha^{101} + \beta^{107} = (-\omega)^{101} + (-\omega)^{107}$$

$$= -[\omega^2 + \omega] = -[-1] = 1$$

94. (a) We have, $\sum_{r=1}^n (x+r-1)(x+r) = 10n$

$$\sum_{r=1}^n (x^2 + xr + (r-1)x + r^2 - r) = 10n$$

$$\Rightarrow \sum_{r=1}^n (x^2 + (2r-1)x + r(r-1)) = 10n$$

$$\Rightarrow nx^2 + \{1+3+5+\dots+(2n-1)\}x + \{1.2+2.3+\dots+(n-1)n\} = 10n$$

$$\Rightarrow nx^2 + n^2x + \frac{(n-1)n(n+1)}{3} = 10n$$

$$\Rightarrow x^2 + nx + \frac{n^2-31}{3} = 0$$

Let α and $\alpha + 1$ be its two solutions

(\(\because\) it has two consecutive integral solutions)

$$\Rightarrow \alpha + (\alpha + 1) = -n$$

$$\Rightarrow \alpha = \frac{-n-1}{2} \quad \dots (i)$$

$$\text{Also } \alpha(\alpha+1) = \frac{n^2-31}{3} \quad \dots (ii)$$

Putting value of (i) in (ii), we get

$$-\left(\frac{n+1}{2}\right)\left(\frac{1-n}{2}\right) = \frac{n^2-31}{3}$$

$$\Rightarrow n^2 = 121 \Rightarrow n = 11$$

95. (c) $(x-1)(x^2+5x-50) = 0$

$$\Rightarrow (x-1)(x+10)(x-5) = 0$$

$$\Rightarrow x = 1, 5, -10$$

$$\text{Sum} = -4$$

96. (c) Let $p(x) = ax^2 + bx + c$

$$\therefore p(0) = 1 \Rightarrow c = 1$$

Also, $p(1) = 4$ & $p(-1) = 6$

$$\Rightarrow a + b + 1 = 4 \text{ \& } a - b + 1 = 6$$

$$\Rightarrow a + b = 3 \text{ \& } a - b = 5$$

$$\Rightarrow a = 4 \text{ \& } b = -1$$

$$p(x) = 4x^2 - x + 1$$

$$p(b) = 16 - 2 + 1 = 15$$

$$p(-2) = 16 + 2 + 1 = 19$$

97. (c) $(x^2 - 5x + 5)^{x^2 + 4x - 60} = 1$

Case I

$x^2 - 5x + 5 = 1$ and $x^2 + 4x - 60$ can be any real number

$$\Rightarrow x = 1, 4$$

Case II

$x^2 - 5x + 5 = -1$ and $x^2 + 4x - 60$ has to be an even number

$$\Rightarrow x = 2, 3$$

where 3 is rejected because for $x = 3$,

$x^2 + 4x - 60$ is odd.

Case III

$x^2 - 5x + 5$ can be any real number and

$$x^2 + 4x - 60 = 0$$

$$\Rightarrow x = -10, 6$$

\Rightarrow Sum of all values of x

$$= -10 + 6 + 2 + 1 + 4 = 3$$

98. (a) $\sqrt{2x+1} - \sqrt{2x-1} = 1$ (i)

$$\Rightarrow 2x + 1 + 2x - 1 - 2\sqrt{4x^2 - 1} = 1$$

$$\Rightarrow 4x - 1 = 2\sqrt{4x^2 - 1}$$

$$\Rightarrow 16x^2 - 8x + 1 = 16x^2 - 4$$

$$\Rightarrow 8x = 5$$

$$\Rightarrow x = \frac{5}{8} \text{ which satisfies equation (i)}$$

$$\text{So, } \sqrt{4x^2 - 1} = \frac{3}{4}$$

99. (a) $\alpha, \beta = \frac{6 \pm \sqrt{36+8}}{2} = 3 \pm \sqrt{11}$

$$\alpha = 3 + \sqrt{11}, \beta = 3 - \sqrt{11}$$

$$\therefore a_n = (3 + \sqrt{11})^n - (3 - \sqrt{11})^n \frac{a_{10} - 2a_8}{2a_9}$$

$$= \frac{(3 + \sqrt{11})^{10} - (3 - \sqrt{11})^{10} - 2(3 + \sqrt{11})^8 + 2(3 - \sqrt{11})^8}{2[(3 + \sqrt{11})^9 - (3 - \sqrt{11})^9]}$$

$$= \frac{(3 + \sqrt{11})^8 [(3 + \sqrt{11})^2 - 2] + (3 - \sqrt{11})^8 [2 - (3 - \sqrt{11})^2]}{2[(3 + \sqrt{11})^9 - (3 - \sqrt{11})^9]}$$

$$= \frac{(3 + \sqrt{11})^8 (9 + 11 + 6\sqrt{11} - 2) + (3 - \sqrt{11})^8 (2 - 9 - 11 + 6\sqrt{11})}{2[(3 + \sqrt{11})^9 - (3 - \sqrt{11})^9]}$$

$$= \frac{6(3 + \sqrt{11})^9 - 6(3 - \sqrt{11})^9}{2[(3 + \sqrt{11})^9 - (3 - \sqrt{11})^9]} = \frac{6}{2} = 3$$

100. (b) $(a-1)(x^4 + x^2 + 1) + (a+1)(x^2 + x + 1)^2 = 0$

$$\Rightarrow (a-1)(x^2 + x + 1)(x^2 - x + 1) + (a+1)(x^2 + x + 1)^2 = 0$$

$$\Rightarrow (x^2 + x + 1)[(a-1)(x^2 - x + 1) + (a+1)(x^2 + x + 1)] = 0$$

$$\Rightarrow (x^2 + x + 1)(ax^2 + x + a) = 0$$

For roots to be distinct and real, $a \neq 0$ and $1 - 4a^2 > 0$

$$\Rightarrow a \neq 0 \text{ and } a^2 < \frac{1}{4}$$

$$\Rightarrow a \in \left(-\frac{1}{2}, 0\right) \cup \left(0, \frac{1}{2}\right)$$

101. (b) $\alpha = 2 + 3i; \beta = 2 - 3i, \gamma = ?$

$$\alpha\beta\gamma = \frac{13}{2} \left[\text{since product of roots} = \frac{d}{a} \right]$$

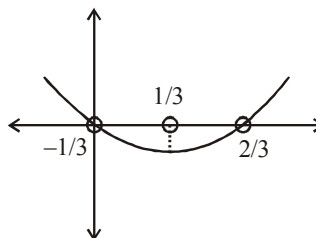
$$\Rightarrow (4+9)\gamma = \frac{13}{2} \Rightarrow \gamma = \frac{1}{2}$$

102. (c) Consider $-3(x - [x])^2 + 2[x - [x]] + a^2 = 0$

$$\Rightarrow 3\{x\}^2 - 2\{x\} - a^2 = 0 \quad (\because x - [x] = \{x\})$$

$$\Rightarrow 3\left(\{x\}^2 - \frac{2}{3}\{x\}\right) = a^2, a \neq 0$$

$$\Rightarrow a^2 = 3\{x\}\left(\{x\} - \frac{2}{3}\right)$$



Now, $\{x\} \in (0, 1)$ and $\frac{-2}{3} \leq a^2 < 1$ (by graph)

Since, x is not an integer

$$\therefore a \in (-1, 1) - \{0\}$$

$$\Rightarrow a \in (-1, 0) \cup (0, 1)$$

103. (a) Consider $\sqrt{3x^2 + x + 5} = x - 3$

Squaring both the sides, we get

$$3x^2 + x + 5 = (x - 3)^2$$

$$\Rightarrow 3x^2 + x + 5 = x^2 + 9 - 6x$$

$$\Rightarrow 2x^2 + 7x - 4 = 0$$

$$\Rightarrow 2x^2 + 8x - x - 4 = 0$$

$$\Rightarrow 2x(x+4) - 1(x+4) = 0$$

$$\Rightarrow x = \frac{1}{2} \text{ or } x = -4$$

For $x = \frac{1}{2}$ and $x = -4$

L.H.S. \neq R.H.S. of equation, $\sqrt{3x^2 + x + 5} = x - 3$
 Also, for every $x \in R$, LHS \neq RHS of the given equation.
 \therefore Given equation has no solution.

104. (c) $x^2 + |2x - 3| - 4 = 0$

$$|2x - 3| = \begin{cases} (2x - 3) & \text{if } x > \frac{3}{2} \\ -(2x - 3) & \text{if } x < \frac{3}{2} \end{cases}$$

for $x > \frac{3}{2}$, $x^2 + 2x - 3 - 4 = 0$

$$x^2 + 2x - 7 = 0$$

$$x = \frac{-2 \pm \sqrt{4 + 28}}{2} = \frac{-2 \pm 4\sqrt{2}}{2} = -1 \pm 2\sqrt{2}$$

Here $x = 2\sqrt{2} - 1 \left\{ 2\sqrt{2} - 1 < \frac{3}{2} \right\}$

for $x < \frac{3}{2}$

$$x^2 - 2x + 3 - 4 = 0$$

$$\Rightarrow x^2 - 2x - 1 = 0$$

$$\Rightarrow x = \frac{2 \pm \sqrt{4 + 4}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$$

Here $x = 1 - \sqrt{2} \left\{ (1 - \sqrt{2}) < \frac{3}{2} \right\}$

Sum of roots : $(2\sqrt{2} - 1) + (1 - \sqrt{2}) = \sqrt{2}$

105. (d) $x^2 - 4\sqrt{2}kx + 2e^{4 \ln k} - 1 = 0$

or, $x^2 - 4\sqrt{2}kx + 2k^4 - 1 = 0$

$\alpha + \beta = 4\sqrt{2}k$ and $\alpha\beta = 2k^4 - 1$

Squaring both sides, we get

$$(\alpha + \beta)^2 = (4\sqrt{2}k)^2 \Rightarrow \alpha^2 + \beta^2 + 2\alpha\beta = 32k^2$$

$$66 + 2\alpha\beta = 32k^2$$

$$66 + 2(2k^4 - 1) = 32k^2$$

$$66 + 4k^4 - 2 = 32k^2 \Rightarrow 4k^4 - 32k^2 + 64 = 0$$

or, $k^4 - 8k^2 + 16 = 0 \Rightarrow (k^2)^2 - 8k^2 + 16 = 0$

$$\Rightarrow (k^2 - 4)(k^2 - 4) = 0 \Rightarrow k^2 = 4, k^2 = 4$$

$$\Rightarrow k = \pm 2$$

Now, $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$

$$\therefore \alpha^3 + \beta^3 = (4\sqrt{2}k) [66 - (2k^4 - 1)]$$

Putting $k = -2$, ($k = +2$ cannot be taken because it does not satisfy the above equation)

$$\therefore \alpha^3 + \beta^3 = (4\sqrt{2}(-2))[66 - 2(-2)^4 - 1]$$

$$\alpha^3 + \beta^3 = (-8\sqrt{2})(66 - 32 + 1) = (-8\sqrt{2}) \quad (35)$$

$$\therefore \alpha^3 + \beta^3 = -280\sqrt{2}$$

106. (a) Let $\frac{1}{\sqrt{\alpha}}$ and $\frac{1}{\sqrt{\beta}}$ be the roots of $ax^2 + bx + 1 = 0$

$$\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = \left(\frac{\sqrt{\alpha} + \sqrt{\beta}}{\sqrt{\alpha\beta}} \right) = -\frac{b}{a}$$

$$\frac{1}{\sqrt{\alpha}\sqrt{\beta}} = \frac{1}{a} \Rightarrow a = \sqrt{\alpha\beta}$$

$$b = -(\sqrt{\alpha} + \sqrt{\beta})$$

$$x(x + b^3) + (a^3 - 3abx) = 0$$

$$\Rightarrow x^2 + (b^3 - 3ab)x + a^3 = 0$$

Putting values of a and b , we get

$$x^2 + \left[(-\sqrt{\alpha} + \sqrt{\beta})^3 + 3(\sqrt{\alpha\beta})(\sqrt{\alpha} + \sqrt{\beta}) \right] + (\alpha\beta)^{3/2} = 0$$

$$\Rightarrow x^2 - [\alpha^{3/2} + \beta^{3/2} + 3\sqrt{\alpha\beta}(\sqrt{\alpha} + \sqrt{\beta}) - 3\sqrt{\alpha\beta}(\sqrt{\alpha} + \sqrt{\beta})]x + (\alpha\beta)^{3/2} = 0$$

$$\Rightarrow x^2 - (\alpha^{3/2} + \beta^{3/2})x + \alpha^{3/2}\beta^{3/2} = 0$$

Roots of this equation are $\alpha^{3/2}, \beta^{3/2}$

107. (b) Given $\alpha^3 + \beta^3 = -p$ and $\alpha\beta = q$

Let $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$ be the root of required quadratic equation.

$$\text{So, } \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta} = \frac{-p}{q}$$

$$\text{and } \frac{\alpha^2}{\beta} \times \frac{\beta^2}{\alpha} = \alpha\beta = q$$

Hence, required quadratic equation is

$$x^2 - \left(\frac{-p}{q} \right)x + q = 0$$

$$\Rightarrow x^2 + \frac{p}{q}x + q = 0 \Rightarrow qx^2 + px + q^2 = 0$$

108. (c) Given quadratic eqn. is

$$x^2 + px + \frac{3p}{4} = 0$$

$$\text{So, } \alpha + \beta = -p, \alpha\beta = \frac{3p}{4}$$

$$\text{Now, given } |\alpha - \beta| = \sqrt{10}$$

$$\Rightarrow \alpha - \beta = \pm\sqrt{10}$$

$$\Rightarrow (\alpha - \beta)^2 = 10 \Rightarrow \alpha^2 + \beta^2 - 2\alpha\beta = 10$$

$$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = 10$$

$$\Rightarrow p^2 - 4 \times \frac{3p}{4} = 10 \Rightarrow p^2 - 3p - 10 = 0$$

$$\Rightarrow p = -2, 5 \Rightarrow p \in \{-2, 5\}$$

109. (c) Given equation is

$$z + \sqrt{2}|z+1| + i = 0$$

put $z = x + iy$ in the given equation.

$$(x + iy) + \sqrt{2}|x + iy + 1| + i = 0$$

$$\Rightarrow x + iy + \sqrt{2} \left[\sqrt{(x+1)^2 + y^2} \right] + i = 0$$

Now, equating real and imaginary part, we get

$$x + \sqrt{2} \sqrt{(x+1)^2 + y^2} = 0 \text{ and}$$

$$y + 1 = 0 \Rightarrow y = -1$$

$$\Rightarrow x + \sqrt{2} \sqrt{(x+1)^2 + (-1)^2} = 0 \quad (\because y = -1)$$

$$\Rightarrow \sqrt{2} \sqrt{(x+1)^2 + 1} = -x$$

$$\Rightarrow 2[(x+1)^2 + 1] = x^2$$

$$\Rightarrow x^2 + 4x + 4 = 0$$

$$\Rightarrow x = -2$$

$$\text{Thus, } z = -2 + i(-1) \Rightarrow |z| = \sqrt{5}$$

110. (c) Given quadratic equation is

$$px^2 + qx + r = 0 \quad \dots(i)$$

$$D = q^2 - 4pr$$

Since α and β are two complex root

$$\therefore \beta = \bar{\alpha} \Rightarrow |\beta| = |\bar{\alpha}| \Rightarrow |\beta| = |\alpha| \quad (\because |\bar{\alpha}| = |\alpha|)$$

Consider

$$|\alpha| + |\beta| = |\alpha| + |\alpha| \quad (\because |\beta| = |\alpha|)$$

$$= 2|\alpha| > 2.1 = 2 \quad (\because |\alpha| > 1)$$

Hence, $|\alpha| + |\beta|$ is greater than 2.

111. (d) Given equation is

$$x^2 - (\sin\alpha - 2)x - (1 + \sin\alpha) = 0$$

Let x_1 and x_2 be two roots of quadratic equation.

$$\therefore x_1 + x_2 = \sin\alpha - 2 \text{ and } x_1x_2 = -(1 + \sin\alpha)$$

$$(x_1 + x_2)^2 = (\sin\alpha - 2)^2 = \sin^2\alpha + 4 - 4\sin\alpha$$

$$\Rightarrow x_1^2 + x_2^2 = \sin^2\alpha + 4 - 4\sin\alpha - 2x_1x_2$$

$$= \sin^2\alpha + 4 - 4\sin\alpha + 2(1 + \sin\alpha)$$

$$= \sin^2\alpha - 2\sin\alpha + 6 \quad \dots(i)$$

Now, By putting

$$\alpha = \frac{\pi}{6}, \alpha = \frac{\pi}{4}, \alpha = \frac{\pi}{3} \text{ and } \alpha = \frac{\pi}{2} \text{ in (i) one by one}$$

We get least value of $x_1^2 + x_2^2$ at $\frac{\pi}{2}$

$$\text{Hence, } \alpha = \frac{\pi}{2}$$

112. (b) $(k-2)x^2 + 8x + k + 4 = 0$

If real roots then,

$$8^2 - 4(k-2)(k+4) > 0$$

$$\Rightarrow k^2 + 2k - 8 < 16$$

$$\Rightarrow k^2 + 6k - 4k - 24 < 0$$

$$\Rightarrow (k+6)(k-4) < 0$$

$$\Rightarrow -6 < k < 4$$

If both roots are negative

then $\alpha\beta$ is +ve

$$\Rightarrow \frac{k+4}{k-2} > 0 \Rightarrow k > -4$$

$$\text{Also, } \frac{k-2}{k+4} > 0 \Rightarrow k > 2$$

Roots are real so, $-6 < k < 4$

So, 6 and 4 are not correct.

Since, $k > 2$, so 1 is also not correct value of k .

$$\therefore k = 3$$

113. (d) $p(x) = 0$

$$\Rightarrow f(x) = g(x)$$

$$\Rightarrow ax^2 + bx + c = a_1x^2 + b_1x + c_1$$

$$\Rightarrow (a - a_1)x^2 + (b - b_1)x + (c - c_1) = 0.$$

It has only one solution, $x = -1$

$$\Rightarrow b - b_1 = a - a_1 + c - c_1 \quad \dots(i)$$

$$\text{Sum of roots } \frac{-(b - b_1)}{(a - a_1)} = -1 - 1$$

$$\Rightarrow \frac{b - b_1}{2(a - a_1)} = 1$$

$$\Rightarrow b - b_1 = 2(a - a_1) \quad \dots(ii)$$

Now $p(-2) = 2$

$$\Rightarrow f(-2) - g(-2) = 2$$

$$\Rightarrow 4a - 2b + c - 4a_1 + 2b_1 - c_1 = 2$$

$$\Rightarrow 4(a - a_1) - 2(b - b_1) + (c - c_1) = 2 \quad \dots(iii)$$

From equations, (i), (ii) and (iii)

$$a - a_1 = c - c_1 = \frac{1}{2}(b - b_1) = 2$$

Now, $p(2) = f(2) - g(2)$
 $= 4(a - a_1) + 2(b - b_1) + (c - c_1)$
 $= 8 + 8 + 2 = 18$

114. (a) Let the correct equation be

$$ax^2 + bx + c = 0$$

Now, Sachin's equation

$$ax^2 + bx + c' = 0$$

Given that, roots found by Sachin's are 4 and 3

$$\Rightarrow -\frac{b}{a} = 7 \quad \dots(i)$$

Rahul's equation, $ax^2 + b'x + c = 0$

Given that roots found by Rahul's are 3 and 2

$$\Rightarrow \frac{c}{a} = 6 \quad \dots(ii)$$

From (i) and (ii), roots of the correct equation

$$x^2 - 7x + 6 = 0 \text{ are } 6 \text{ and } 1.$$

115. (c) Since both the roots of given quadratic equation lie in the line $Re z = 1$ i.e., $x = 1$, hence real part of both the roots are 1.

Let both roots be $1 + i\alpha$ and $1 - i\alpha$

Product of the roots, $1 + \alpha^2 = \beta$

$$\because \alpha^2 + 1 \geq 1$$

$$\therefore \beta \geq 1 \Rightarrow \therefore \beta \in (1, \infty)$$

116. (b) $x^2 - x + 1 = 0 \Rightarrow x = \frac{1 \pm \sqrt{1-4}}{2}$

$$x = \frac{1 \pm \sqrt{3}i}{2}$$

$$\alpha = \frac{1}{2} + i\frac{\sqrt{3}}{2} = -\omega^2$$

$$\beta = \frac{1}{2} - i\frac{\sqrt{3}}{2} = -\omega$$

$$\alpha^{2009} + \beta^{2009} = (-\omega^2)^{2009} + (-\omega)^{2009}$$

$$= -\omega^2 - \omega = 1$$

$$[\because \omega^3 = 1]$$

117. (b) Given that roots of the equation

$bx^2 + cx + a = 0$ are imaginary

$$\therefore c^2 - 4ab < 0 \quad \dots(i)$$

Let $y = 3b^2x^2 + 6bcx + 2c^2$

$$\Rightarrow 3b^2x^2 + 6bcx + 2c^2 - y = 0$$

As x is real, $D \geq 0$

$$\Rightarrow 36b^2c^2 - 12b^2(2c^2 - y) \geq 0$$

$$\Rightarrow 12b^2(3c^2 - 2c^2 + y) \geq 0 \quad [\because b^2 \geq 0]$$

$$\Rightarrow c^2 + y \geq 0 \Rightarrow y \geq -c^2$$

But from eqn. (i), $c^2 < 4ab$ or $-c^2 > -4ab$

$$\therefore \text{we get } y \geq -c^2 > -4ab$$

$$\Rightarrow y > -4ab$$

118. (c) Let α and β are roots of the equation

$$x^2 + ax + 1 = 0$$

$$\alpha + \beta = -a \text{ and } \alpha\beta = 1$$

$$\text{Given that } |\alpha - \beta| < \sqrt{5}$$

$$\Rightarrow \sqrt{(\alpha + \beta)^2 - 4\alpha\beta} < \sqrt{5}$$

$$(\because (\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta)$$

$$\Rightarrow \sqrt{a^2 - 4} < \sqrt{5} \Rightarrow a^2 - 4 < 5$$

$$\Rightarrow a^2 - 9 < 0 \Rightarrow a^2 < 9 \Rightarrow -3 < a < 3$$

$$\Rightarrow a \in (-3, 3)$$

119. (c) Given equation is $x^2 - 2mx + m^2 - 1 = 0$

$$\Rightarrow (x - m)^2 - 1 = 0$$

$$\Rightarrow (x - m + 1)(x - m - 1) = 0$$

$$\Rightarrow x = m - 1, m + 1$$

$$m - 1 > -2 \text{ and } m + 1 < 4$$

$$\Rightarrow m > -1 \text{ and } m < 3 \Rightarrow -1 < m < 3$$

120. (b) Given that $x^2 + px + q = 0$

$$\text{Sum of roots} = \tan 30^\circ + \tan 15^\circ = -p$$

$$\text{Product of roots} = \tan 30^\circ \cdot \tan 15^\circ = q$$

$$\tan 45^\circ = \frac{\tan 30^\circ + \tan 15^\circ}{1 - \tan 30^\circ \tan 15^\circ} \Rightarrow \frac{-p}{1 - q} = 1$$

$$\Rightarrow -p = 1 - q \Rightarrow q - p = 1$$

$$\therefore 2 + q - p = 3$$

121. (d) $z^2 + z + 1 = 0 \Rightarrow z = \omega \text{ or } \omega^2$

$$\text{So, } z + \frac{1}{z} = \omega + \omega^2 = -1$$

$$\left[\because \frac{1}{z} = \omega^2 \text{ and } 1 + \omega + \omega^2 = 0 \right]$$

$$z^2 + \frac{1}{z^2} = \omega^2 + \omega = -1,$$

$$[\because \omega^3 = 1]$$

$$z^3 + \frac{1}{z^3} = \omega^3 + \omega^3 = 2$$

$$z^4 + \frac{1}{z^4} = -1, \quad z^5 + \frac{1}{z^5} = -1$$

$$\text{and } z^6 + \frac{1}{z^6} = 2$$

$$\therefore \text{The given sum} = 1+1+4+1+1+4 = 12$$

122. (b) $\tan\left(\frac{P}{2}\right), \tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0$

$$\tan\left(\frac{P}{2}\right) + \tan\left(\frac{Q}{2}\right) = -\frac{b}{a}$$

$$\tan\left(\frac{P}{2}\right) \cdot \tan\left(\frac{Q}{2}\right) = \frac{c}{a}$$

$$\frac{\tan\left(\frac{P}{2}\right) + \tan\left(\frac{Q}{2}\right)}{1 - \tan\left(\frac{P}{2}\right)\tan\left(\frac{Q}{2}\right)} = \tan\left(\frac{P}{2} + \frac{Q}{2}\right) = 1$$

$$\left[\because P+Q = \frac{\pi}{2} \right]$$

$$\Rightarrow \frac{-\frac{b}{a}}{1 - \frac{c}{a}} = 1 \Rightarrow -\frac{b}{a} = \frac{a-c}{a}$$

$$\Rightarrow -b = a - c \Rightarrow c = a + b$$

123. (d) Let $\alpha, \alpha + 1$ be roots

Then $\alpha + \alpha + 1 = b = \text{sum of roots}$

$\alpha(\alpha + 1) = c = \text{product of roots}$

$$\therefore b^2 - 4c = (2\alpha + 1)^2 - 4\alpha(\alpha + 1) = 1.$$

124. (d) Given that 4 is a root of $x^2 + px + 12 = 0$

$$\Rightarrow 16 + 4p + 12 = 0 \Rightarrow p = -7$$

Now, the equation $x^2 + px + q = 0$

has equal roots.

$$\therefore D = 0$$

$$\Rightarrow p^2 - 4q = 0 \Rightarrow q = \frac{p^2}{4} = \frac{49}{4}$$

125. (c) Let the second root be α .

Then $\alpha + (1 - p) = -p \Rightarrow \alpha = -1$

Also $\alpha(1 - p) = 1 - p$

$$\Rightarrow (\alpha - 1)(1 - p) = 0 \Rightarrow p = 1 \quad [\because \alpha = -1]$$

$$\therefore \text{Roots are } \alpha = -1 \text{ and } 1 - p = 0$$

126. (c) Given that

$$x^2 - 3|x| + 2 = 0 \Rightarrow |x|^2 - 3|x| + 2 = 0$$

$$\Rightarrow (|x| - 2)(|x| - 1) = 0$$

$$\Rightarrow |x| = 1, 2 \Rightarrow x = \pm 1, \pm 2$$

$$\therefore \text{No. of solution} = 4$$

127. (b) Let one roots of given equation be α

\therefore Second roots be 2α then

$$\alpha + 2\alpha = 3\alpha = \frac{1 - 3a}{a^2 - 5a + 3}$$

$$\Rightarrow \alpha = \frac{1 - 3a}{3(a^2 - 5a + 3)} \quad \dots(i)$$

$$\text{and } \alpha \cdot 2\alpha = 2\alpha^2 = \frac{2}{a^2 - 5a + 3}$$

$$\therefore 2 \left[\frac{1}{9} \frac{(1 - 3a)^2}{(a^2 - 5a + 3)^2} \right] = \frac{2}{a^2 - 5a + 3}$$

[from (i)]

$$\frac{(1 - 3a)^2}{(a^2 - 5a + 3)} = 9$$

$$\Rightarrow 9a^2 - 6a + 1 = 9a^2 - 45a + 27$$

$$\Rightarrow 39a = 26 \Rightarrow a = \frac{2}{3}$$

128. (d) Given that $Z^2 + aZ + b = 0$;

$$Z_1 + Z_2 = -a \text{ \& } Z_1 Z_2 = b$$

$0, Z_1, Z_2$ form an equilateral triangle

$$\therefore 0^2 + Z_1^2 + Z_2^2 = 0, Z_1 + Z_1 Z_2 + Z_2 \cdot 0$$

(for an equilateral triangle,

$$Z_1^2 + Z_2^2 + Z_3^2 = Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1)$$

$$\Rightarrow Z_1^2 + Z_2^2 = Z_1 Z_2$$

$$\Rightarrow (Z_1 + Z_2)^2 = 3Z_1 Z_2$$

$$\therefore a^2 = 3b$$

129. (a) $p + q = -p \Rightarrow q = 2p$

and $pq = q \Rightarrow q(p - 1) = 0$

$$\Rightarrow q = 0 \text{ or } p = 1.$$

If $q = 0$, then $p = 0$.

or $p = 1$, then $q = -2$.

130. (a) Product of real roots = $\frac{c}{a} = \frac{9}{t^2} > 0, \forall t \in R$

\therefore Product of real roots is always positive.

131. (a) Let α and β are roots of the equation $x^2 + ax + b = 0$ and γ and δ be the roots of the equation $x^2 + bx + a = 0$ respectively.

$$\therefore \alpha + \beta = -a, \alpha\beta = b \text{ and } \gamma + \delta = -b, \gamma\delta = a.$$

$$\text{Given } |\alpha - \beta| = |\gamma - \delta| \Rightarrow (\alpha - \beta)^2 = (\gamma - \delta)^2$$

$$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = (\gamma + \delta)^2 - 4\gamma\delta$$

$$\Rightarrow a^2 - 4b = b^2 - 4a$$

$$\Rightarrow (a^2 - b^2) + 4(a - b) = 0$$

$$\Rightarrow a + b + 4 = 0 \quad (\because a \neq b)$$

132. (a) Given that $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$;

$\Rightarrow \alpha$ & β are roots of equation, $x^2 = 5x - 3$

$$\text{or } x^2 - 5x + 3 = 0$$

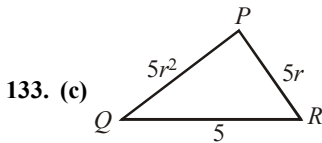
$$\therefore \alpha + \beta = 5 \text{ and } \alpha\beta = 3$$

Thus, the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is

$$x^2 - x\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) + \frac{\alpha\beta}{\alpha\beta} = 0$$

$$\Rightarrow x^2 - x\left(\frac{\alpha^2 + \beta^2}{\alpha\beta}\right) + 1 = 0$$

$$\text{or } 3x^2 - 19x + 3 = 0$$



ΔPQR is possible if

$$5 + 5r > 5r^2$$

$$\Rightarrow 1 + r > r^2$$

$$\Rightarrow r^2 - r - 1 < 0$$

$$\Rightarrow \left(r - \frac{1}{2} + \frac{\sqrt{5}}{2}\right)\left(r - \frac{1}{2} - \frac{\sqrt{5}}{2}\right) < 0$$

$$\Rightarrow r \in \left(\frac{-\sqrt{5} + 1}{2}, \frac{\sqrt{5} + 1}{2}\right)$$

$$\therefore \frac{7}{4} \notin \left(\frac{-\sqrt{5} + 1}{2}, \frac{\sqrt{5} + 1}{2}\right) \therefore r \neq \frac{7}{4}$$

134. (a) $ax^2 - 2bx + 5 = 0$,

If α and α are roots of equations, then sum of roots

$$2\alpha = \frac{2b}{a} \Rightarrow \alpha = \frac{b}{a}$$

$$\text{and product of roots} = \alpha^2 = \frac{5}{a} \Rightarrow \frac{b^2}{a^2} = \frac{5}{a}$$

$$\Rightarrow b^2 = 5a \quad (a \neq 0) \quad \dots(i)$$

$$\text{For } x^2 - 2bx - 10 = 0$$

$$\alpha + \beta = 2b \quad \dots(ii)$$

$$\text{and } \alpha\beta = -10 \quad \dots(iii)$$

$$\alpha = \frac{b}{a} \text{ is also root of } x^2 - 2bx - 10 = 0$$

$$\Rightarrow b^2 - 2ab^2 - 10a^2 = 0$$

$$\text{By eqn. (i)} \Rightarrow 5a - 10a^2 - 10a^2 = 0$$

$$\Rightarrow 20a^2 = 5a$$

$$\Rightarrow a = \frac{1}{4} \text{ and } b^2 = \frac{5}{4}$$

$$\alpha^2 = 20 \text{ and } \beta^2 = 5$$

$$\text{Now, } \alpha^2 + \beta^2 = 5 + 20 = 25$$

135. (b) Let, the roots of the equation, $x^2 + (2 - \lambda)x + (10 - \lambda) = 0$ are α and β .

Also roots of the given equation are

$$\frac{\lambda - 2 \pm \sqrt{4 - 4\lambda + \lambda^2 - 40 + 4\lambda}}{2} = \frac{\lambda - 2 \pm \sqrt{\lambda^2 - 36}}{2}$$

The magnitude of the difference of the roots is $|\sqrt{\lambda^2 - 36}|$

$$\text{So, } \alpha^3 + \beta^3 = \frac{(\lambda - 2)^3}{4} + \frac{3(\lambda - 2)(\lambda^2 - 36)}{4}$$

$$= \frac{(\lambda - 2)(4\lambda^2 - 4\lambda - 104)}{4} = (\lambda - 2)(\lambda^2 - \lambda - 26) = f(\lambda)$$

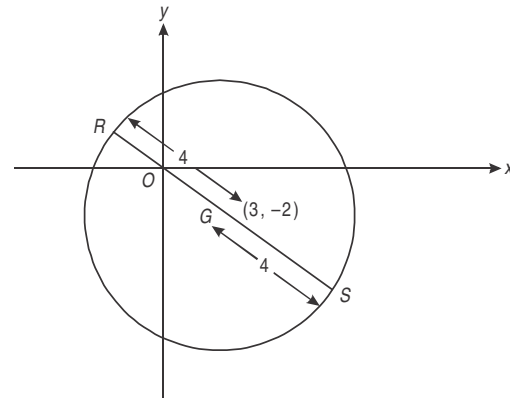
As $f(\lambda)$ attains its minimum value at $\lambda = 4$.

Therefore, the magnitude of the difference of the roots is

$$|i\sqrt{20}| = 2\sqrt{5}$$

136. (b) $|z - (3 - 2i)| \leq 4$ represents a circle whose centre is $(3, -2)$ and radius = 4.

$|z| = |z - 0|$ represents the distance of point 'z' from origin $(0, 0)$



Suppose RS is the normal of the circle passing through origin 'O' and G is its center $(3, -2)$.

Here, OR is the least distance

and OS is the greatest distance

$$OR = RG - OG \text{ and } OS = OG + GS \quad \dots(i)$$

As, $RG = GS = 4$

$$OG = \sqrt{3^2 + (-2)^2} = \sqrt{9+4} = \sqrt{13}$$

From (i), $OR = 4 - \sqrt{13}$ and $OS = 4 + \sqrt{13}$

$$\begin{aligned} \text{So, required difference} &= (4 + \sqrt{13}) - (4 - \sqrt{13}) \\ &= \sqrt{13} + \sqrt{13} = 2\sqrt{13} \end{aligned}$$

137. (c) $x^2 + bx - 1 = 0$ common root

$$x^2 + x + b = 0$$

$$\begin{array}{r} - \quad - \quad - \\ x = \frac{b+1}{b-1} \end{array}$$

Put $x = \frac{b+1}{b-1}$ in equation

$$\left(\frac{b+1}{b-1}\right)^2 + \left(\frac{b+1}{b-1}\right) + b = 0$$

$$(b+1)^2 + (b+1)(b-1) + b(b-1)^2 = 0$$

$$b^2 + 1 + 2b + b^2 - 1 + b(b^2 - 2b + 1) = 0$$

$$2b^2 + 2b + b^3 - 2b^2 + b = 0$$

$$b^3 + 3b = 0$$

$$b(b^2 + 3) = 0$$

$$b^2 = -3$$

$$b = \pm\sqrt{3}i$$

$$|b| = \sqrt{3}$$

138. (d) We have

$$f(x) = x^2 + 2bx + 2c^2$$

$$\text{and } g(x) = -x^2 - 2cx + b^2, (x \in R)$$

$$\Rightarrow f(x) = (x+b)^2 + 2c^2 - b^2$$

$$\text{and } g(x) = -(x+c)^2 + b^2 + c^2$$

$$\text{Now, } f_{\min} = 2c^2 - b^2 \text{ and } g_{\max} = b^2 + c^2$$

$$\text{Given : } \min f(x) > \max g(x)$$

$$\Rightarrow 2c^2 - b^2 > b^2 + c^2$$

$$\Rightarrow c^2 > 2b^2$$

$$\Rightarrow |c| > |b|\sqrt{2}$$

$$\Rightarrow \left|\frac{c}{b}\right| > \sqrt{2} \Rightarrow \left|\frac{c}{b}\right| > \sqrt{2}$$

$$\Rightarrow \left|\frac{c}{b}\right| \in (\sqrt{2}, \infty).$$

139. (b) Let α, β be the common roots of both the equations.

For first equation $ax^2 + bx + c = 0$, we have

$$\alpha + \beta = \frac{-b}{a} \quad \dots(i)$$

$$\alpha \cdot \beta = \frac{c}{a} \quad \dots(ii)$$

For second equation $2x^2 + 3x + 4 = 0$, we have

$$\alpha + \beta = \frac{-3}{2} \quad \dots(iii)$$

$$\alpha \cdot \beta = \frac{2}{1} \quad \dots(iv)$$

Now, from (i) & (iii) & from (ii) & (iv)

$$\frac{-b}{a} = \frac{-3}{2} \quad \quad \quad \frac{c}{a} = \frac{2}{1}$$

$$\frac{b}{a} = \frac{3/2}{1}$$

Therefore on comparing we get $a = 1, b = \frac{3}{2}$ & $c = 2$

putting these values in first equation, we get

$$x^2 + \frac{3}{2}x + 2 = 0 \text{ or } 2x^2 + 3x + 4 = 0$$

from this, we get $a = 2, b = 3; c = 4$

or $a : b : c = 2 : 3 : 4$

140. (a) Given equations are

$$x^2 + 2x + 3 = 0 \quad \dots(i)$$

$$ax^2 + bx + c = 0 \quad \dots(ii)$$

Roots of equation (i) are imaginary roots in order pair.

According to the question (ii) will also have both roots same as (i). Thus

$$\frac{a}{1} = \frac{b}{2} = \frac{c}{3} = \lambda \text{ (say)}$$

$$\Rightarrow a = \lambda, b = 2\lambda, c = 3\lambda$$

Hence, required ratio is $1 : 2 : 3$

141. (a) $\frac{x-5}{x^2+5x-14} > 0 \Rightarrow x^2+5x-14 < x-5$

$$\Rightarrow x^2+4x-9 < 0$$

$$\Rightarrow \alpha = -5, -4, -3, -2, -1, 0, 1$$

$\alpha = -5$ does not satisfy any of the options

$\alpha = -4$ satisfy the option (a) $\alpha^2 + 3\alpha - 4 = 0$

142. (c) $x^2 - (a+1)x + a^2 + a - 8 = 0$

Since roots are different, therefore $D > 0$

$$\Rightarrow (a+1)^2 - 4(a^2 + a - 8) > 0$$

$$\Rightarrow (a-3)(3a+1) < 0$$

There are two cases arises.

Case I. $a-3 > 0$ and $3a+1 < 0$

$$\Rightarrow a > 3 \text{ and } a < -\frac{11}{3}$$

Hence, no solution in this case

Case II : $a-3 < 0$ and $3a+1 > 0$

$$\Rightarrow a < 3 \text{ and } a > -\frac{11}{3}$$

$$\therefore -\frac{11}{3} < a < 3 \Rightarrow -2 < a < 3$$

143. (a) Given that $\left| z - \frac{4}{z} \right| = 2$

$$|z| = \left| z - \frac{4}{z} + \frac{4}{z} \right| \leq \left| z - \frac{4}{z} \right| + \frac{4}{|z|}$$

$$\Rightarrow |z| \leq 2 + \frac{4}{|z|}$$

$$\Rightarrow |z|^2 - 2|z| - 4 \leq 0$$

$$\Rightarrow \left(|z| - \frac{2 + \sqrt{20}}{2} \right) \left(|z| - \frac{2 - \sqrt{20}}{2} \right) \leq 0$$

$$\Rightarrow (|z| - (1 + \sqrt{5})) (|z| - (1 - \sqrt{5})) \leq 0$$

$$\begin{array}{c} + \quad \quad - \quad \quad + \\ -\infty \quad | \quad | \quad | \quad \infty \\ (1 - \sqrt{5}) \quad (1 + \sqrt{5}) \end{array}$$

$$\Rightarrow (-\sqrt{5} + 1) \leq |z| \leq (\sqrt{5} + 1)$$

$$\Rightarrow |z|_{\max} = \sqrt{5} + 1$$

144. (d) Let the roots of equation $x^2 - 6x + a = 0$ be α and 4β and that of the equation

$x^2 - cx + 6 = 0$ be α and 3β . Then

$$\alpha + 4\beta = 6 \quad \dots(i) \quad 4\alpha\beta = a \quad \dots(ii)$$

$$\text{and } \alpha + 3\beta = c \quad \dots(iii) \quad 3\alpha\beta = 6 \quad \dots(iv)$$

$$\Rightarrow a = 8 \text{ (from (ii) and (iv))}$$

\therefore The equation becomes $x^2 - 6x + 8 = 0$

$$\Rightarrow (x-2)(x-4) = 0$$

\Rightarrow roots are 2 and 4

$\Rightarrow \alpha = 2, \beta = 1 \therefore$ Common root is 2.

145. (b) $y = \frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$

$$3x^2(y-1) + 9x(y-1) + 7y - 17 = 0$$

$$D \geq 0 \quad \because x \text{ is real}$$

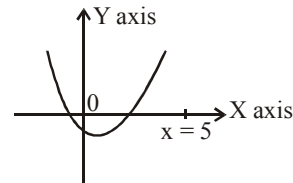
$$81(y-1)^2 - 4 \times 3(y-1)(7y-17) \geq 0$$

$$\Rightarrow (y-1)(y-41) \leq 0 \Rightarrow 1 \leq y \leq 41$$

\therefore Max value of y is 41

$$\begin{array}{c} + \quad \quad - \quad \quad + \\ -8 \quad | \quad | \quad | \quad 8 \\ \quad \quad 1 \quad \quad 41 \end{array}$$

146. (c) Given that both roots of quadratic equation are less than 5 then (i)



Discriminant ≥ 0

$$4k^2 - 4(k^2 + k - 5) \geq 0$$

$$4k^2 - 4k^2 - 4k + 20 \geq 0$$

$$4k \leq 20 \Rightarrow k \leq 5$$

(ii) $p(5) > 0$

$$\Rightarrow f(5) > 0; 25 - 10k + k^2 + k - 5 > 0$$

$$\Rightarrow k^2 - 9k + 20 > 0$$

$$\Rightarrow k(k-4) - 5(k-4) > 0$$

$$\Rightarrow (k-5)(k-4) > 0$$

$$\begin{array}{c} + \quad \quad - \quad \quad + \\ -\infty \quad | \quad | \quad | \quad \infty \\ \quad \quad 4 \quad \quad 5 \end{array}$$

$$\Rightarrow k \in (-\infty, 4) \cup (5, \infty)$$

(iii) $\frac{\text{Sum of roots}}{2} < 5$

$$\Rightarrow -\frac{b}{2a} = \frac{2k}{2} < 5$$

$$\Rightarrow k < 5$$

The intersection of (i), (ii) & (iii) gives

$$k \in (-\infty, 4).$$

147. (a) Given equation is $x^2 - (a-2)x - a - 1 = 0$

$$\Rightarrow \alpha + \beta = a - 2; \quad \alpha\beta = -(a+1)$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= a^2 - 2a + 6 = (a-1)^2 + 5$$

For min. value of $\alpha^2 + \beta^2, a - 1 = 0$

$$\Rightarrow a = 1.$$